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Ranking and candidacy probabilities of signal-to-noise ratio random variables under rayleigh fading

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Abstract
In design and analysis problems of wireless communication systems, a set of signal-to-noise ratio random variables may have to be dealt with. For example, this applies to transmit and/or receive diversity systems, and to scheduling and resource allocation problems. In such cases, it can be important to know how a given signal-to-noise ratio random variable ranks among the whole set of random variables. For example, in opportunistic multiuser systems, allocating resources to a user depends strongly on the probability of that user's signal-to-noise ratio being larger than those of all (or most) other users. Considering a set of signal-to-noise ratio random variables, ranking probability is defined in this paper as the probability of a signal-to-noise ratio random variable being smaller than a given number of signal-to-noise ratio random variables in the set. Candidacy probability is defined as the probability of a signal-to-noise ratio random variable belonging to a subset of signal-to-noise ratio random variables with highest values. Closed form expressions for ranking and candidacy probabilities of signal-to-noise ratio random variables are derived, assuming Rayleigh fading. The derived probabilities are compared to those found by simulation. Comparisons confirm the correctness of the derived expressions. Simulations have been performed assuming a cellular base station that is serving a number of users. In several experiments, different levels of variation in the average signal-to-noise ratios of the users have been assumed. Results confirm the need for applying some fairness constraints in signal-to-noise ratio order-based scheduling algorithms to prevent situations where some users may not get enough access to system resources.

Keywords Wireless communications · Vehicular communications · Opportunistic communications · Rayleigh fading

1 Introduction

Wireless spectrum is a scarce resource, and improving the efficiency of spectrum utilization is crucial, especially for providing high-rate-data services [1]. This includes the ability to support high speed data transmission with different quality of service (QoS) requirements. Channel conditions experienced by different users of a communication system have generally different and statistically independent random time variation patterns [2]. In other words, every user of the system has its own sequence of channel conditions, when measured over some period of time. Since achievable throughput is a function of the received signal-to-noise ratio (SNR), users cannot be expected to all achieve the same instantaneous throughput [3].

When using adaptive modulation, the channel state can be estimated and made available to the transmitter [4, 5]. The transmitter modulation and coding scheme (MCS) can then be adapted relative to the channel state, such that the transmission power and symbol rate are adjusted based on the channel condition.

Scheduling involves the use of a rule, or set of rules, to specify which user is allowed to transmit/receive during a time slot [6]. Scheduling can play important roles in meeting user QoS requirements, such as throughput, delay, and fairness [7]. In opportunistic-type wireless communication systems, transmissions from/to users are scheduled based

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on their time-varying channel conditions, prioritizing users with better channels, and therefore, resulting in bit error rate improvements [8–10]. When transmitting/receiving users are the ones with better channel conditions, substantial gains in total network throughput can be achieved, compared to cases where channel state information is not utilized for purposes of scheduling transmissions [11]. When throughput is increased, spectral efficiency is increased as well. Actually, opportunistic scheduling can increase the spectral efficiency by a factor of two or more [9, 10].

The challenge in using opportunistic-type scheduling is that transmissions from/to users with generally bad channel conditions (e.g., users that are far away from the base station (BS) in a cellular system) are sacrificed. Therefore, the scheduling scheme should exploit the channel conditions to achieve higher utilization of wireless resources, while achieving good levels of fairness and satisfaction among users [3]. A recent example of applying this concept to scheduling in a vehicular communication network is provided in [12].

Proportional fairness (PF) scheduling [13] is a popular technique that prioritizes users with better channel conditions, but without necessarily assigning system resources to the ones with best channel conditions. This increases achievable throughput and allows for some level of fairness in assigning resources to users. The priority of a user being assigned a system resource is obtained by dividing the achievable data rate of the user by averaged achieved data rate of the user [14]. For the sake of reaching different tradeoffs between achievable throughout, fairness and computational complexity, many variations of the PF algorithm, as well as other new algorithms that are based on it have appeared in recent literature [15–17]. A PF scheduler that takes into account channel non-stationarity for moving cars in a vehicular network is proposed in [18].

1.1 Paper motivation

In wireless communication systems, the concept of electing a candidate subset of users based on their channel quality has been used earlier in downlink scheduling scenarios [19, 20]. In some scheduling techniques, the candidate set of users is chosen from the subset of users that have achieved the lowest throughputs [21, 22].

By scheduling transmissions to users in the set of candidates taking into account their already achieved throughputs, the scheduler seeks to create fairness in assigning resources to users. This motivates efforts to derive closed-form expressions of user candidacy probabilities, as defined below. The obtained expressions can then be used to determine the performance of scheduling algorithms that operate subject to fairness constraints.

Let’s consider a set of SNR random variables. The ranking probability of an SNR random variable in the set is defined as the probability of the random variable being smaller than a given number of other random variables in the set. The candidacy probability is defined as the probability of an SNR random variable belonging to a subset of (candidate) SNR random variables with highest values. Let’s consider a scheduling algorithm that tries to achieve fairness in assigning system resources to users by performing scheduling in two steps. In the first step, a subset of users having the highest instantaneous SNRs among all users requesting system resources is created. This is the subset of candidate users. In the second step, the one candidate user with the lowest already achieved throughput is assigned system resources in proportion to its achievable throughput. To determine the probability of assigning system resources to any given user, it is necessary to determine that user’s probability of being a candidate user. This obviously requires first determining the ranking probabilities of the user.

1.2 Paper contributions

The contributions of this work are as follows:

- Derivation of a closed form expression for ranking probabilities of SNR random variables in a cellular wireless communication system under Rayleigh fading. The derived expression can be useful in the analysis and design of SNR order-based scheduling problems. In such problems, it can be important to determine the probability of a given user SNR occupying a certain rank (order) within the set of all user SNRs. This a typical problem in rank order statistics [23]. To the best of knowledge of the authors, the ranking probability formulas that are derived in this article have not been published in any earlier research articles.
- Derivation of a closed form expression for candidacy probabilities of SNR random variables in a cellular wireless communication system under Rayleigh fading. The derived expression can be useful in the analysis and design of SNR order-based scheduling problems. In such problems, it can be important to determine the probability of a given user SNR belonging to a set of candidate user SNRs that is a subset of the set of all user SNRs. To the best of knowledge of the authors, the candidacy probability formulas that are derived in this article have not been published in any earlier research articles.
- Verification of the correctness of the derived expressions using computer simulation. This is done by comparing the ranking and candidacy probabilities that are calculated using the derived expressions to their counterparts that are determined through computer simulations.

1.3 Paper organization

The remainder of this paper is organized as follows. Ranking and candidacy probabilities are defined and derived in Sect. 2. Results and comparisons are presented in Sect. 3. Finally, Sect. 4 concludes the paper.

2 Ranking and candidacy probabilities

2.1 Definitions

2.1.1 Ranking probability

In a set of U SNR random variables $\{\gamma_u\}_{u=1}^{U}$, random variable γ_u has ranking k, where $1 \leq k \leq U$, when there are exactly $k-1$ SNR random variables with larger values than γ_u. The notation $P_k(\gamma_u)$ will be used to denote the probability of γ_u having a ranking k. The set of probabilities $\{P_k(\gamma_u)\}_{k=1}^{U}$ specify the ranking probability mass function (RPMF) of γ_u.

2.1.2 Candidacy probability

Let $\mathbb{K}$ denote the subset of K (out of U) largest SNR random variables. The candidacy probability of γ_u is its probability of being an element of $\mathbb{K}$, i.e., $\Pr(\gamma_u \in \mathbb{K})$. This probability will be denoted as $Q_K(\gamma_u)$. Note that when $\gamma_u \in \mathbb{K}$, then the ranking of γ_u cannot exceed K. Hence, $Q_K(\gamma_u)$ specifies the value of the ranking cumulative distribution function (RCDF) of γ_u.

2.2 Derivation of ranking and candidacy probabilities

Consider a set of U statistically independent exponentially-distributed SNR random variables $\{\gamma_u\}_{u=1}^{U}$. Such random variables can be encountered in a multiuser wireless communication system operating in Rayleigh fading. Let $\mathbb{U} = \{u\}_{u=1}^{U}$ be the set of SNR indices. Let $\mathbb{U}_i = \mathbb{U} - \{i\}$ be the set of indices, excluding i, where $i \in \mathbb{U}$. Obviously, set $\mathbb{U}_i$ has $U-1$ elements, as opposed to set $\mathbb{U}$ having U elements. Let the $U \times 1$ vector $\underline{j}$ represent a unique permutation of the U elements of $\mathbb{U}$. The $U!$ possible permutation vectors of $\mathbb{U}$ will be combined into set $\mathbb{J}$, whose elements take the form $\underline{j} = \left[\, j_1 \;\, j_2 \;\cdots\; j_U \,\right]^T$, where $j_m \in \mathbb{U}$ for $m = 1, 2, \cdots, U$; with $j_m \neq j_n$ for $m \neq n$.

2.2.1 Derivation of ranking probabilities

Assuming communication over a Rayleigh fading channel, a received SNR random variable γ has an exponential probability density function of the form

$$f_\gamma(\gamma) = \alpha e^{-\alpha\gamma}, \tag{1}$$

where $\alpha = 1/\bar{\gamma}$ and $\bar{\gamma}$ is the mean value of the SNR. Let's define the event $A_{\underline{j}}$ such that it occurs whenever $\gamma_{j_1} > \gamma_{j_2} > \cdots > \gamma_{j_U}$, i.e.,

$$A_{\underline{j}} = \left\{ \underline{j} | \gamma_{j_1} > \gamma_{j_2} > \cdots > \gamma_{j_U} \right\}. \tag{2}$$

The probability of $A_{\underline{j}}$ occurring can be determined by integrating the joint probability density function (PDF) of the U SNR random variables over the region of values in (2). Due to the independence of the SNR random variables, the joint PDF is the product of U functions like the one in (1), each representing the PDF of one of the SNR random variables. Based on the above, the joint PDF of the U SNR random variables can be written in the form

$$\begin{aligned} &f_{\gamma_1\gamma_2\ldots\gamma_U}(\gamma_1, \gamma_2, \ldots, \gamma_U) \\ &\quad = \alpha_1\alpha_2\ldots\alpha_U \; e^{-\alpha_1\gamma_1-\alpha_2\gamma_2-\ldots-\alpha_U\gamma_U}. \end{aligned} \tag{3}$$

Using the joint PDF in (3), the probability of event $A_{\underline{j}}$, defined in (2), can be calculated by first integrating the PDF over γ_{j_1} from γ_{j_2} to infinity. The following step is to integrate the PDF over γ_{j_2} from γ_{j_3} to infinity. Similar steps have to be performed until the PDF is integrated over $\gamma_{j_{U-1}}$ from γ_{j_U} to infinity. The last step is to integrate the PDF over γ_{j_U} from zero to infinity. The above procedure can be compactly represented as in the following equation

$$\begin{aligned} \Pr(A_{\underline{j}}) \;=& \int_0^\infty \int_{x_{j_U}}^\infty \cdots \int_{x_{j_3}}^\infty \int_{x_{j_2}}^\infty \alpha_{j_1} e^{-\alpha_{j_1}x_{j_1}} dx_{j_1} \\ & \alpha_{j_2} e^{-\alpha_{j_2}x_{j_2}} dx_{j_2} \cdots \alpha_{j_U} e^{-\alpha_{j_U}x_{j_U}} dx_{j_U}. \end{aligned} \tag{4}$$

The integration in (4) can be easily done by first integrating over the variable x_{j_1}, then over the variable x_{j_2}, and so on until the final step of integrating over the variable x_{j_U}. It should be noted that lower limit of the x_{j_1} integral in (4) is x_{j_2}. Taking this into account, performing the integration over x_{j_1} yields

$$\begin{aligned} \Pr(A_{\underline{j}}) \;=& \int_0^\infty \int_{x_{j_U}}^\infty \cdots \int_{x_{j_3}}^\infty \alpha_{j_2} e^{-(\alpha_{j_1}+\alpha_{j_2})x_{j_2}} dx_{j_2} \\ & \alpha_{j_3} e^{-\alpha_{j_3}x_{j_3}} dx_{j_3} \cdots \alpha_{j_U} e^{-\alpha_{j_U}x_{j_U}} dx_{j_U}. \end{aligned} \tag{5}$$

Similarly to the previous step, performing the integration in (5) over the variable x_{j_2}, and noting that the lower limit of the x_{j_2} integral in (5) is x_{j_3}, yields

$$\Pr(A_{\underline{j}}) = \frac{\alpha_{j_2}}{\alpha_{j_1}+\alpha_{j_2}} \times \int_0^\infty \int_{x_{j_U}}^\infty \cdots \int_{x_{j_4}}^\infty \alpha_{j_3} e^{-(\alpha_{j_1}+\alpha_{j_2}+\alpha_{j_3})x_{j_3}} dx_{j_3} \alpha_{j_4} e^{-\alpha_{j_4} x_{j_4}} dx_{j_4} \cdots \alpha_{j_U} e^{-\alpha_{j_U} x_{j_U}} dx_{j_U}. \tag{6}$$

Continuing the above procedure until integration over the variable x_{j_U} is performed, the U-fold multiple integration in (4) yields

$$Pr(A_{\underline{j}}) = \frac{\alpha_{j_1}\alpha_{j_2}\cdots\alpha_{j_U}}{\alpha_{j_1}\left(\alpha_{j_1}+\alpha_{j_2}\right)\cdots\left(\alpha_{j_1}+\alpha_{j_2}+\cdots+\alpha_{j_U}\right)}. \tag{7}$$

As a special case of (7), let's consider the case of only two SNR random variables, i.e., $U = 2$, to get the very well-known result

$$Pr(A_{\underline{j}}) = \frac{\alpha_{j_2}}{\alpha_{j_1}+\alpha_{j_2}}. \tag{8}$$

To simplify the expression in (7), let for $l \in \mathbb{U}$,

$$S_{\underline{j},l} = \sum_{m=1}^{l} \alpha_{j_m}. \tag{9}$$

Using (9) in (7), one gets

$$\Pr(A_{\underline{j}}) = \prod_{l=1}^{U} \frac{\alpha_{j_l}}{S_{\underline{j},l}}. \tag{10}$$

Note that

$$\prod_{l=1}^{U} \alpha_{j_l} = \prod_{l=1}^{U} \alpha_l. \tag{11}$$

Using (11), (10) simplifies to

$$\Pr(A_{\underline{j}}) = \prod_{l=1}^{U} \frac{\alpha_l}{S_{\underline{j},l}}. \tag{12}$$

Let $\mathbb{J}^{k,i}$ be the subset of $\mathbb{J}$ with element vectors $\underline{j}^{k,i} = \left[j_1^{k,i} \; j_2^{k,i} \cdots j_U^{k,i} \right]^T$, where $j_l^{k,i} \in \mathbb{U}_i$ for $l \neq k$ and $j_k^{k,i} = i$. SNR random variable γ_i has ranking k for all events $A_{\underline{j}^{k,i}}$ that are associated with vectors $\underline{j}^{k,i}$. Similarly to (12), the probability of $A_{\underline{j}^{k,i}}$ is equal to

$$\Pr(A_{\underline{j}^{k,i}}) = \prod_{l=1}^{U} \frac{\alpha_l}{S_{\underline{j}^{k,i},l}}, \tag{13}$$

where from (9) we have

$$S_{\underline{j}^{k,i},l} = \sum_{m=1}^{l} \alpha_{j_m^{k,i}}. \tag{14}$$

Summing over all the vectors in $\mathbb{J}^{k,i}$, γ_i has ranking k with a probability $P_k(\gamma_i)$ that is given by

$$P_k(\gamma_i) = \sum_{\underline{j}^{k,i} \in \mathbb{J}^{k,i}} \left(\prod_{l=1}^{U} \frac{\alpha_l}{S_{\underline{j}^{k,i},l}} \right). \tag{15}$$

Note that the summation in (15) contains $(U-1)!$ terms. The RPMF of γ_i is therefore equal to

$$P_{\gamma_i}(x) = \sum_{k=1}^{U} P_k(\gamma_i)\delta(x-k). \tag{16}$$

Note that scaling all elements in $\{\alpha_l\}_{l=1}^{U}$ by the same factor does not affect the result in (15).

2.2.2 Derivation of candidacy probabilities

Let $\mathbb{K}$ denote the set of K (out of U) largest SNR random variables. The candidacy probability $Q_K(\gamma_i)$ of γ_i, as defined above, is equal to $\Pr(\gamma_i \in \mathbb{K})$. This probability is equal to

$$Q_K(\gamma_i) = \sum_{k=1}^{K} P_k(\gamma_i). \tag{17}$$

Substituting (15) into (17) yields

$$Q_K(\gamma_i) = \sum_{k=1}^{K} \sum_{\underline{j}^{k,i} \in \mathbb{J}^{k,i}} \left(\prod_{l=1}^{i} \frac{\alpha_l}{S_{\underline{j}^{k,i},l}} \right). \tag{18}$$

3 Verification and results

To verify correctness of the above ranking and candidacy probability expressions, a number of computer simulations with very large numbers of runs have been conducted. A few examples with different SNR distributions are discussed below. In the studied cases, it is not possible to distinguish the theoretical and simulation results; because they have been found to be identical.

In the simulations, we have studied a cellular system consisting of one BS serving U users. We have considered the downlink only; because this is sufficient to illustrate the use of the derived ranking and candidacy probabilities. Similar to what is assumed in many existing works (e.g., [24, 25], the BS has been assumed to apply SNR order-based downlink scheduling. Users are assumed to be located at random locations in the BS coverage area. No restrictions have been imposed on the locations of users in the case of equal average SNRs; because power control is assumed to compensate the average SNR variations. In the cases of small, medium and large average SNR variations, user locations have been specified such that the required level of average SNR variation is satisfied.

Most of the below simulation results have been obtained assuming $U = 8$. Choosing larger values of U would produce results with similar trends, while needing considerably larger simulation times. A single result assuming $U = 10$ is included at the end of this section as an example. Ranking probabilities have been sketched for users 1, 4, 8 (1, 5, 10 in the $U = 10$ case). The reason for focusing on these users is that their average SNRs summarize the whole of SNR average values.

3.1 Equal average SNRs

Consider the special case when all the SNR random variables have the same mean $\bar{\gamma}$, and therefore, the same $\alpha = 1/\bar{\gamma}$. This case applies to scheduling of homogeneous users in a multiple-input multiple-output (MIMO) or massive MIMO (mMIMO) network [26]. Equal average user SNRs are ensured by assuming that the BS applies max-min power control to achieve fair user scheduling in the downlink [27].

Substituting the fixed α into (7), (9), (12) and (13) results in $S_{\underline{j},l} = l\alpha$ and $\Pr(A_{\underline{j}}) = \Pr(A_{\underline{j}^{k,i}}) = 1/U!$ for all l, $\underline{j}$ and k. Substituting the resulting values into (15) yields $P_k(\gamma_i) = 1/U$ for all k and i, which is the expected result. Clearly, the RPMF is equal to the same value of $1/U$ for all k and i. Substituting the equal ranking probabilities into (17) yields an equal candidacy probability $Q_k(\gamma_i) = K/U$ for all k and i.

From the viewpoint of SNR order-based scheduling, all user SNRs have the same ranking probability and, consequently, the same candidacy probability. Hence, simple round robin (RR) scheduling is a natural choice. When used in this case, RR scheduling allows allocating equal system resources to all users (maximum fairness), while resulting in maximum network throughput.

3.2 Small average SNR variations

This case applies to scenarios when the distances between the users have mostly small values. In other words, the users are assumed to be located in a relatively small area; such that the user-BS distance experiences only small variations. Similar scenarios have been considered in studying user grouping and downlink power allocation when non-orthogonal multiple access (NOMA) is used [28].

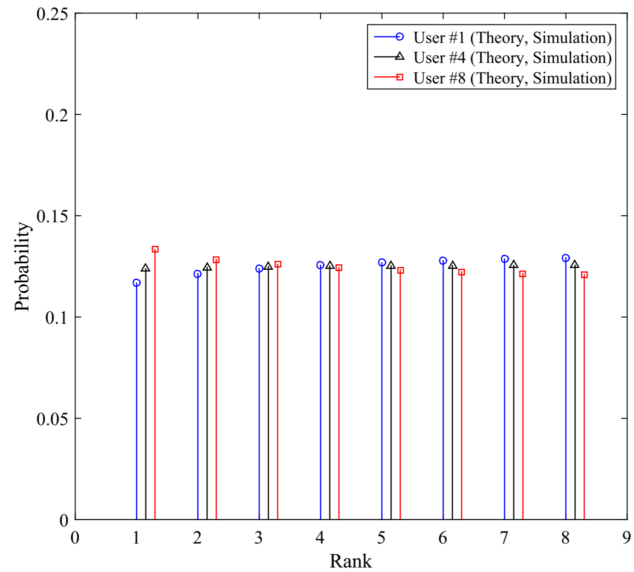

Fig. 1 Ranking PMF for users $i = 1, 4, 8$ with small average SNR variations when $U = 8$

In this case we let $U = 8$, $i = 1, 4, 8$ and $\{\alpha_l\}_{l=1}^{8} = \{1, 1.01, \ldots, 1.07\}$. The ratio of the largest and smallest mean SNR values is equal to $\bar{\gamma}_1/\bar{\gamma}_U = 1.07$; which we consider a small ratio. Figure 1 shows the RPMF given in (15) for users $i = 1, 4, 8$. The main observation here is that the RPMFs are relatively flat, something we would expect when the mean values of the involved random variables are almost equal.

From the viewpoint of SNR order-based scheduling, user SNRs have approximately equal ranking probabilities and, consequently, approximately equal candidacy probabilities. Hence, RR scheduling can be used for the sake of simplicity while tolerating a small throughput reduction. When used in this case, RR scheduling allows allocating equal system resources to all users (maximum fairness), while resulting in close-to-maximum network throughput.

3.3 Medium average SNR variations

This case applies to scenarios when the distances between the users have mostly moderate values. In other words, the users are assumed to be located in a medium-sized area; such that the user-BS distance experiences moderate variations. Similar scenarios have been considered in studying user grouping and downlink power allocation when NOMA is used [28].

In this case we let $U = 8$, $i = 1, 4, 8$ and $\{\alpha_l\}_{l=1}^{8} = \{1, 1.1, \ldots, 1.7\}$. The ratio of the largest and smallest mean SNR values is equal to $\bar{\gamma}_1/\bar{\gamma}_U = 1.7$; which we consider a medium ratio. Figure 2 shows the RPMF given in (15) for users $i = 1, 4, 8$. The main observation here is that the SNR

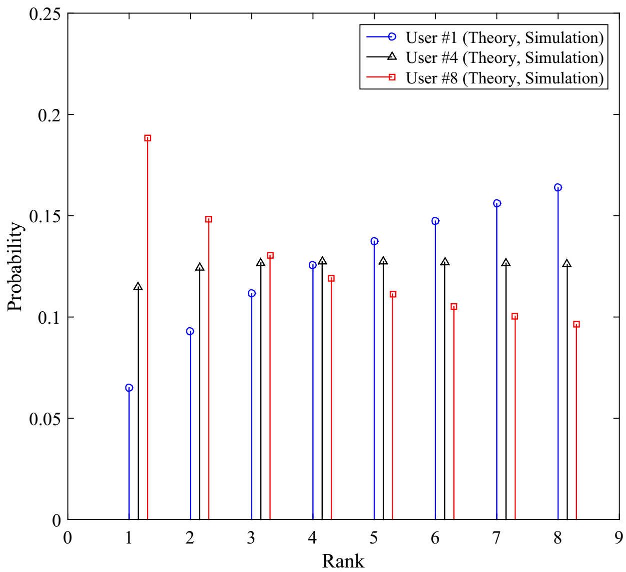

Fig. 2 Ranking PMF for users $i = 1, 4, 8$ with medium average SNR variations when $U = 8$

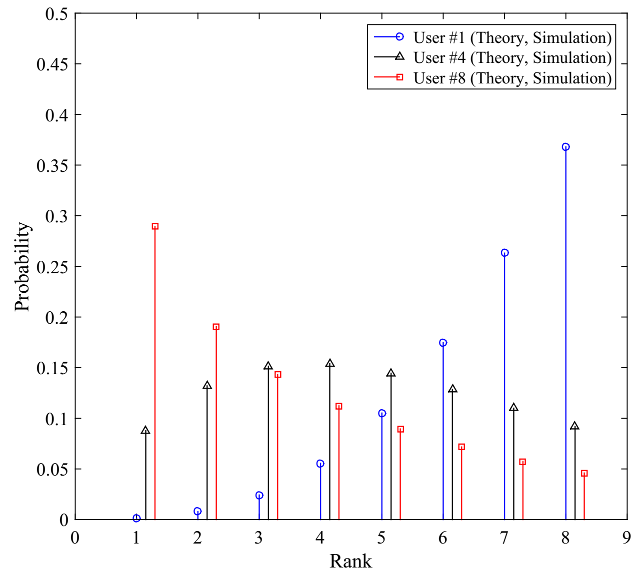

Fig. 3 Ranking PMF for users $i = 1, 4, 8$ with large average SNR variations when $U = 8$

RPMF of the middle user $i = 4$ is relatively flat, meaning that it has almost the same probability of being ranked anywhere within the group of SNRs. In contrast, the SNR RPMFs of the first and last users are, respectively, a slowly increasing and a slowly decreasing functions of the rank. The simple explanation of this behavior is that the first user has a small mean SNR, meaning that the rank of its SNR is more likely to assume higher values. Meanwhile, the last user has a larger mean SNR, meaning that the rank of its SNR is more likely to assume lower values.

From the viewpoint of SNR order-based scheduling, user SNRs have somehow varying ranking probabilities and correspondingly varying candidacy probabilities. Hence, a scheduling algorithm that prioritizes higher SNRs can be used to optimize the throughput. This would involve some unfairness in assigning resources to users; because of the variations in average SNRs. Therefore, the scheduling algorithm has to be designed to ensure an acceptable level of fairness in assigning resources to users.

3.4 Large average SNR variations

This case applies to scenarios when the distances between the users have mostly large values. In other words, the users are assumed to be located in a large area; such that the user-BS distance experiences large variations. Similar scenarios have been considered in studying user grouping and downlink power allocation when NOMA is used [28].

In this case we let $U = 8$, $i = 1, 4, 8$ and $\{\alpha_l\}_{l=1}^{8} = \{1, 2, \ldots, 8\}$. The ratio of the largest and smallest mean SNR values is equal to $\bar{\gamma}_1/\bar{\gamma}_U = 8$; which we consider a large ratio. Figure 3 shows the RPMF given in (15) for users $i = 1, 4, 8$ when $U = 8$, while Fig. 4 shows the RPMF given in (15) for users $i = 1, 5, 10$ when $U = 10$. The main observation here is that the SNR RPMF of the middle user peaks for medium values of the rank, meaning that it has highest probability of being ranked nearly in the middle. In contrast, the SNR RPMFs of the first and last users are, respectively, a steeply increasing and a steeply decreasing functions of the rank. The simple explanation of this behavior is that the first user has a small mean SNR, meaning that the rank of its SNR is strongly more likely to assume higher values. Meanwhile, the last user has a much larger mean SNR, meaning that the rank of its SNR is strongly more likely to assume lower values.

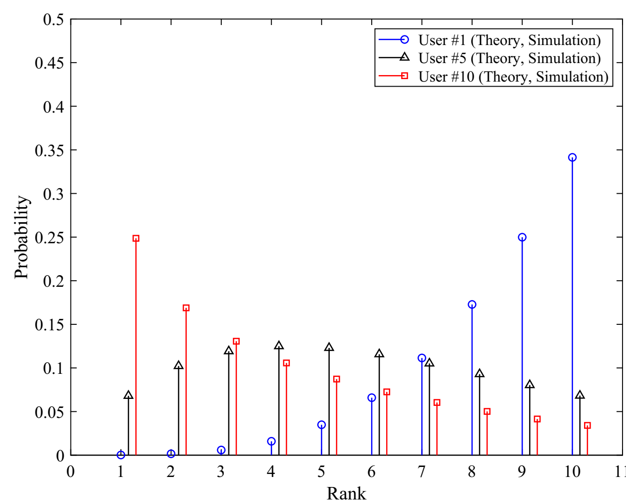

Fig. 4 Ranking PMF for users $i = 1, 5, 10$ with large average SNR variations when $U = 10$

From the viewpoint of SNR order-based scheduling, user SNRs have widely varying ranking probabilities and correspondingly widely candidacy probabilities. Hence, a scheduling algorithm that prioritizes higher SNRs can be used to optimize the throughput. However, this would involve a great deal of unfairness in assigning resources to users; because of the wide variations in average SNRs. Therefore, the scheduling algorithm has to be designed with strict fairness constraints on assigning resources to users.

4 Conclusion and future work

Closed form expressions for ranking and candidacy probabilities of SNR random variables have been derived, assuming Rayleigh fading. The probabilities computed using the derived theoretical expressions have been compared to those found by simulation. Comparisons confirm the correctness of the derived expressions.

A number of computer simulations have been conducted. In a case with small variations in the means of the SNR random variables, the RPMFs have been found to be relatively flat, something we would expect when the mean values of the involved random variables are almost equal. In other cases with medium and large variations in the means of the SNR random variables, the RPMFs have been found to have shapes that are consistent with the distribution of the SNR mean values.

As far as calculating the derived ranking and candidacy probability expressions, it is fair to say that it has been a time-consuming process. Therefore, it is highly recommended to consider reformulating the expressions in ways that can reduce the computational cost of performing the calculations.

Further to the above, this work can be extended in a number of ways. One possible extension is to determine the ranking and candidacy probabilities in the important case of Rician fading, which has applications in small cell systems where a significant line-of-sight component is likely to be present in the received signal. More general fading channel models, like the Nakagami-*m* model can also be studied. Another future research direction is to apply the results of this work to opportunistic scheduling scenarios with constraints that do not limit channel access to only users with best channel conditions. Air time and throughput fairness constraints, as well as quality of service requirements are possible factors in such scenarios.

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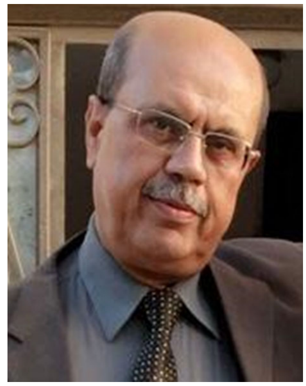
Declarations

Conflicts of interest The authors have not disclosed any competing interests.

References

1. Liu, X., Chong, E.K., Shroff, N.B. (2001). Transmission scheduling for efficient wireless utilization. In: Proceedings IEEE INFOCOM 2001. Conference on Computer Communications. Twentieth Annual Joint Conference of the IEEE Computer and Communications Society (Cat. No. 01CH37213), vol. 2, pp. 776–785. IEEE.
2. Tse, D., & Viswanath, P. (2005). *Fundamentals of Wireless Communication*. Cambridge: Cambridge University Press.
3. Liu, X., Edwin, K., Chong, K. P., & Shroff, N. B. (2001). Opportunistic transmission scheduling with resource-sharing constraints in wireless networks. *IEEE Journal on Selected Areas in Communications, 19*(5), 2053–2064.
4. Goldsmith, A. (2005). *Wireless Communications*. Cambridge: Cambridge University Press.
5. Svensson, A. (2007). An introduction to adaptive QAM modulation schemes for known and predicted channels. *Proceedings of the IEEE, 95*(12), 2322–2336.
6. Asadi, A., & Mancuso, V. (2013). A survey on opportunistic scheduling in wireless communications. *IEEE Communications Surveys & Tutorials, 15*(4), 1671–1688.
7. Zhang, H. (1995). Service disciplines for guaranteed performance service in packet-switching networks. *Proceedings of the IEEE, 83*(10), 1374–1396.
8. Knopp, R., Humblet, P. (1995). Information capacity and power control in single-cell multiuser communications. In: Proceedings of the IEEE International Conference on Communications ICC '95, vol. 1. Seattle, WA, USA, pp. 331–335.
9. Viswanath, P., Tse, D. N. C., & Laroia, R. (2002). Opportunistic beamforming using dumb antennas. *IEEE Transactions on Information Theory, 48*(6), 1277–1294.
10. Liu, X., Chong, E.K.P., Shroff, N.B. (2003). Optimal opportunistic scheduling in wireless networks. In: Proceedings of the 2003 IEEE 58th Vehicular Technology Conference. VTC 2003-Fall, vol. 3. Orlando, FL, USA, pp. 1417–1421.
11. Viswanath, P., Tse, D. N. C., & Laroia, R. (2002). Opportunistic beamforming using dumb antennas. *IEEE Transactions on Information Theory, 48*(6), 1277–1294.
12. Zheng, K., Liu, F., Zheng, Q., Xiang, W., & Wang, W. (2013). A graph-based cooperative scheduling scheme for vehicular networks. *IEEE Transactions on Vehicular Technology, 62*(4), 1450–1458.
13. Jalali, A., Padovani, R., Pankaj, R. (2000). Data throughput of CDMA-HDR a high efficiency-high data rate personal communication wireless system. In: Proceedings of the 2000 IEEE 51st Vehicular Technology Conference. VTC 2000-Spring, vol. 3. Tokyo, Japan, pp. 1854–1858.
14. Mansouri, W., Ali, K.B., Zarai, F., Obaidat, M.S. (2015). Radio resource management for heterogeneous wireless networks: Schemes and simulation analysis. In: Modeling and Simulation of Computer Networks and Systems, pp. 767–792. Elsevier.
15. Hadzi-Velkov, Z., Nikoloska, I., Chingoska, H., & Zlatanov, N. (2016). Proportional fair scheduling in wireless networks with rf energy harvesting and processing cost. *IEEE Communications Letters, 20*(10), 2107–2110.
16. Cheng, Y., Fu, P., Ding, Y., Li, B., & Yuan, X. (2017). Proportional fairness in cognitive wireless powered communication networks. *IEEE Communications Letters, 21*(6), 1397–1400.
17. Li, X., Shankaran, R., Orgun, M. A., Fang, G., & Xu, Y. (2018). Resource allocation for underlay D2D communication with proportional fairness. *IEEE Transactions on Vehicular Technology, 67*(7), 6244–6258.
18. Nguyen, T.T.N., Brun, O., Prabhu, B.J. (2019). An algorithm for improved proportional-fair utility for vehicular users. In:

International Conference on Analytical and Stochastic Modeling Techniques and Applications, pp. 115–130. Springer
19. Liu, F., Mähönen, P., Petrova, M. (2016). Proportional fairness-based power allocation and user set selection for downlink noma systems. In: 2016 IEEE International Conference on Communications (ICC), pp. 1–6. IEEE.
20. Al-Wani, M.M., Sali, A., Ali, B.M., Salah, A.A., Navaie, K., Leow, C.Y., Noordin, N.K., Hashim, S.J. (2019). On short term fairness and throughput of user clustering for downlink non-orthogonal multiple access system. In: 2019 IEEE 89th Vehicular Technology Conference (VTC2019-Spring), pp. 1–6. IEEE
21. Banat, M.M., Shatnawi, R. (2021) Round opportunistic fair downlink scheduling in wireless communications networks. In: Proceedings of the 1st International Congress on Engineering Technologies: EngiTek 2020, 16-18 June 2020, Irbid, Jordan, p. 116. CRC Press.
22. Banat, M.M., Shatnawi, R.F. (2022). Round opportunistic fair wireless downlink scheduling with performance guarantees. In: 2022 13th International Conference on Information and Communication Systems (ICICS), pp. 448–45. IEEE
23. Wolfe, D. A. (2012). *Nonparametrics: Statistical Methods Based on Ranks and Its Impact on the Field of Nonparametric Statistics* (pp. 1101–1110). Boston, MA: Springer. https://doi.org/10.1007/978-1-4614-1412-4_96
24. Morsi, R., Michalopoulos, D. S., & Schober, R. (2015). Multiuser scheduling schemes for simultaneous wireless information and power transfer over fading channels. *IEEE Transactions on Wireless Communications, 14*(4), 1967–1982.
25. Almekhlafi, M., Arfaoui, M.A., Elhattab, M., Assi, C., Ghrayeb, A. (2021). Joint scheduling of embb and urllc services in ris-aided downlink cellular networks. In: 2021 International Conference on Computer Communications and Networks (ICCCN), pp. 1–9. IEEE.
26. Zhang, Z., Sun, Y., Sabharwal, A., Chen, Z. (2018). Impact of channel state misreporting on multi-user massive mimo scheduling performance. In: IEEE INFOCOM 2018-IEEE Conference on Computer Communications, pp. 917–925. IEEE
27. Pawar, A. R., Kashyap, S., & Chouhan, S. (2021). Impact of max-min power control, channel estimation and user grouping strategies on uplink massive mimo-noma systems. *IEEE Transactions on Vehicular Technology, 70*(8), 7858–7869.
28. Dommel, J., Faehse, S., Thiele, L. (2017). On power allocation and user grouping for sparse coded non-orthogonal multiple access in the downlink. In: 2017 IEEE 86th Vehicular Technology Conference (VTC-Fall), pp. 1–4. IEEE



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