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Simulation of Doubly Selective Fading Channels Using Autoregressive Modeling and Optimum Receiver Design

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Abstract—**Autoregressive (AR) modeling is a well known scheme for signal and system modeling. In this paper, we investigate using AR models to generate doubly selective fading with correlated and uncorrelated paths. We briefly consider an autoregressive moving average (ARMA) extension. AR models are shown to closely approximate channel correlation functions for reasonable model orders. When we use the AR/ARMA-generated channel taps in the receiver to replace channel estimation, the sacrifice in receiver performance is shown to be insignificant. The AR/ARMA approaches can be used to reduce complexity at the receiver side by using low-complexity filters.**

Keywords- Doubly Selective; Autoregressive; Autocorrelation; Rake Receiver.

I. Introduction

Talking about new generations of mobile technology, especially third generation services and applications, there is a demand for high data rates and for high speed mobility. These two aspects have substantial impact on the channel, through frequency selectivity and time selectivity, respectively. This situation is commonly referred to as fading channel double selectivity.

Wireless fading channels are typically modeled in the literature as having either Rayleigh or Rician signal envelope statistics [1]. The Rician probability distribution model is used when a line of site (LOS) received signal component is present [2]. There exist many algorithms in the literature to generate Rayleigh and Rician channel variates [3][4]. These algorithms are based commonly on three main approaches: the sum of sinusoids (SOS) approach [3], the inverse discrete Fourier transform (IDFT) approach [4] and the autoregressive process (AR) approach [5]. There have been observed significant problems with the first two approaches above for fading channel simulation. The SOS approach, based on the classical Jakes fading simulator, produces fading signals that lack the wide sense stationarity (WSS) property [6]. The IDFT is a block-based algorithm. Therefore, it cannot be used to accurately describe a time selective channel (recall that this is a fast fading channel). Additionally, the IDFT approach has high storage requirements [6]. The AR approach can generate a high quality fading channel that accurately satisfies the desired statistical channel properties with low storage requirements [5].

The above-mentioned approaches are suitable, to varying degrees, for general-purpose fading channel simulation. However, modeling doubly selective fading channels is more challenging. In this type of channels, the model has to describe the temporal evolution of the channel. This behavior is missing in the SoS and the IDFT fading channel models. The AR model has the advantage of following the channel time variations [7].

The remainder of this paper is organized as follows. In Section II, the AR and ARMA models are defined, and a few related issues are discussed. In Section III, the proposed receiver, based on the AR/ARMA models, is explained. Comparisons based on computer simulations are presented in Section IV. Performance of the rake receiver using the proposed techniques is presented in Section V. The paper is concluded in section VI.

II. AR/ARMA Fading Channel Modeling

A. Uncorrelated Scattering

The channel input/output relationship can be given by:

$$y(n)=\sum_{l=0}^{L-1} h(n,l)x(n-l)+w(n) \qquad (1)$$

where, $x(n)$ and $y(n)$ are the input and output signals of the channel, respectively, and $w(n)$ is a zero-mean complex-valued additive white Gaussian noise process. In the AR model, the l-th channel tap is described as the output of the AR filter shown in Figure 1.

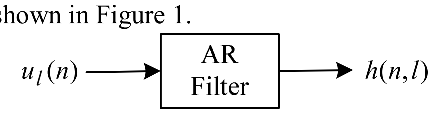

Figure 1: Autoregressive fading model of the l-th channel tap

where the AR filter has a transfer function given by:

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$$G_l(z) = \frac{1}{1 - \sum_{p=1}^{P} a_l(p) z^{-p}} \qquad (2)$$

Using the AR model, the channel time-varying impulse response can be written in the form

$$h(n,l) = u_l(n) * Z^{-1}\left\{ \frac{1}{1 - \sum_{p=1}^{P} a_l(p) z^{-p}} \right\} \qquad (3)$$

where $Z^{-1}\{.\}$ denotes taking the inverse Z transform. Equivalently, we could use the convolution formula,

$$h(n,l) = \sum_{p=1}^{P} a_l(p) h(n-p,l) + u_l(n) \qquad (4)$$

where $\{a_l(p)\}_{p=1}^{P}$ are the model coefficients and $u_l(n)$ is a zero mean complex Gaussian process with

$$E\left[u_l(n).u_l^*(n-k)\right] \overset{\Delta}{=} \sigma_{u,l}^2.\delta(k).\delta(l-l') \qquad (5)$$

Under the assumption of wide-sense stationary uncorrelated scattering (WSSUS), the fading generation process can be assumed to be the same for all taps $\{h(n,l)\}_{l=0}^{L-1}$. The AR model provides potential for simulating fading channels with arbitrary correlation functions. To see this, it should be emphasized that the channel correlation function appears explicitly in the Yule-Walker form of the impulse response AR process. Let's multiply both sides of (4) by $h^*(n-k,l)$ and apply statistical expectation to both sides.

For $k = 0,1,\cdots,P$, this results in

$$r_h(k,l) = \sum_{p=1}^{P} a_l(p) r_h(k-p,l) + \sigma_{u,l}^2 \delta(k) \qquad (6)$$

where $r_h(k,l)$, is the discrete-time counterpart of the channel correlation function defined in [8], and $\sigma_{u,l}^2$ is the variance of $u_l(n)$. We can rewrite (6) in matrix format for $k = 0,1,\cdots,P$ as

$$R'_h \underline{a}' = \sigma_{u,l}^2 \underline{e}_1 \qquad (7)$$

$$\text{Where } R'_h = \begin{bmatrix} r_h(0,l) & r_h^*(1,l) & \cdots & r_h^*(P,l) \\ r_h(1,l) & r_h(0,l) & \cdots & r_h^*(P-1,l) \\ \vdots & \cdots & \ddots & \vdots \\ r_h(P,l) & \cdots & \cdots & r_h(0,l) \end{bmatrix} \qquad (8)$$

$$\underline{a}' = \begin{bmatrix} 1 & -a_l(1) & \cdots & -a_l(P) \end{bmatrix}^T \qquad (9)$$

$$\underline{e}_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T \qquad (10)$$

The above Yule walker equations can be written as in [9] in the alternative form

$$R_h \underline{a} = -\underline{v} \qquad (11)$$

where,

$$R_h = \begin{bmatrix} r_h(0,l) & r_h(-1,l) & \cdots & r_h(-P+1,l) \\ r_h(1,l) & r_h(0,l) & \cdots & r_h(-P+2,l) \\ \vdots & \vdots & \ddots & \vdots \\ r_h(P-1,l) & r_h(P-2,l) & \cdots & r_h(0,l) \end{bmatrix} \qquad (12)$$

$$\underline{a} = \begin{bmatrix} a_l(1) & a_l(2) & \cdots & a_l(P) \end{bmatrix}^T \qquad (13)$$

$$\underline{v} = \begin{bmatrix} r_h(1,l) & r_h(2,l) & \cdots & r_h(P,l) \end{bmatrix}^T \qquad (14)$$

The white noise source variance is given by:

$$\sigma_{u,l}^2 = r_h(0,l) + \sum_{k=1}^{P} a_k r_h(-k,l) \qquad (15)$$

In order to solve for the unknowns $\{a_l(p)\}_{p=1}^{P}$ and $\sigma_{u,l}^2$, the correlations $\{r_h(k,l)\}_{k=0}^{P}$ have to be known, and the model order P has to be specified. The selection of P should be based on a trade-off between the required accuracy and the resulting complexity. Another channel characteristic that should be specified is the channel power delay profile, given by:

$$r_h(k,l) = \beta_l^2 J_0(2\pi f_{\max} kT) \qquad (16)$$

Hence, to generate the l-th tap of a Rayleigh fading channel, we need to solve (11) for the set of unknowns (the AR model coefficients) $\{a_l(1),\cdots,a_l(p)\}$. After the coefficients have been determined, (15) can be used to determine the white noise source variance $\sigma_{u,l}^2$. It should be pointed out that the channel correlations for lags $k = 1,2\cdots,P$ should be available before the above can be achieved. This requires knowledge of the following set of parameters:

- The normalized Doppler spread f_m, and more specifically, the product $f_m T$
- The model order P
- The coefficients of the power delay profile corresponding to the l-th tap, β_l^2 (refer to (16))

Once these parameters have been specified, (11) can be solved either directly, or through the Levinson-Durbin recursion algorithm. It was shown in [5] that the condition number of the matrix R_h is inversely proportional to the model order. Hence, if the AR model order P is increased, numerical instability problems cannot be avoided. Numerical problems in the solution typically yield unstable AR filters. A solution to this problem was suggested in [5]. The idea is to replace the channel correlation $r_h(k,l)$ by $r_h(k,l)+\varepsilon$, where ε is a small positive quantity with typical values as in TABLE [5]. This is equivalent to adding a white noise with variance ε to the main process $u_l(n)$. Using this small modification, the AR model can be used to generate a fading channel of arbitrary order P .

TABLE I: Order of ε observed to yield the most accurately correlated AR simulator outputs for the $J_0(.)$ ACF model

f_m	ε
0.001	10^{-5}
0.005	10^{-6}
0.01	10^{-7}
0.05	10^{-8}

A majority of published papers that propose methods for simulating fading channels use the land mobile channel model. We will use the same assumption in this paper.

B. Correlated Scattering

Starting from a number of uncorrelated Rayleigh paths, their correlated counterparts can be generated through a matrix operation. Note that we are dealing with correlated paths not correlated samples of a single path.

To illustrate the above procedure, let $[X_1 \quad X_2 \quad X_3]$ be three uncorrelated equal power Rayleigh paths, which are generated independently. Then, the operation

$$\begin{bmatrix} Y1 \\ Y2 \\ Y3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} \quad (17)$$

can generate three correlated Rayleigh paths $[Y_1 \quad Y_2 \quad Y_3]$. Matrix elements $\{C_{ij}\}$ in (17) depend on the required covariance matrix of the correlated paths. In this paper, we use the covariance matrix [10]

$$C = \begin{bmatrix} 1 & 0.9 & 0.5 \\ 0.9 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{bmatrix} \quad (18)$$

C. The ARMA Approach

In this section a proposed scheme for enhancing the performance of the channel generation is presented. The AR model used so far in the paper will be generalized into an autoregressive moving average (ARMA) model. A theoretical background will first be provided. Let's assume that some white noise $u(n)$ is to be filtered using a linear shift-invariant filter having a rational system function that is given by:

$$G(z) = \frac{B_q(z)}{A_p(z)} = \frac{\sum_{k=0}^{q} b_q(k) z^{-k}}{1 - \sum_{k=1}^{p} a_p(k) z^{-k}} \quad (19)$$

Note that $G(z)$ has p poles and q zeros. The output process $x(n)$ is wide-sense stationary. Given a power spectral density of the white noise equal to $P_u(z) = \sigma_u^2$, the power spectral density of $x(n)$ will be

$$P_x(z) = \sigma_v^2 \frac{B_q(z) B_q^*(1/z^*)}{A_p(z) A_p^*(1/z^*)} \quad (20)$$

The power spectral density can also be written in terms of the frequency variable ω in the form

$$P_x(e^{j\omega}) = \sigma_v^2 \frac{\left|B_q(e^{j\omega})\right|^2}{\left|A_p(e^{jw})\right|^2} \quad (21)$$

The above equation represents the power spectrum of an ARMA process of order (p,q), which will be denoted as ARMA(p,q). The relationship between $x(n)$ and $v(n)$ can be expressed using the linear constant coefficient difference equation [9]:

$$x(n) + \sum_{l=1}^{P} a_p(l) x(n-l) = \sum_{l=0}^{q} b_q(l) u(n-l) \quad (22)$$

Multiplying both sides of (22) by $x^*(n-k)$ and taking the expected value,

$$r_x(k) + \sum_{l=1}^{P} a_p(l) r_x(k-l) = \sum_{l=0}^{q} b_q(l) \mathrm{E}\left[u(n-l) x^*(n-k)\right] \quad (23)$$

Since $u(n)$ is WSS, then $u(n)$ and $x(n)$ are jointly WSS [9] and

$$E\left\{u(n-l) x^*(n-k)\right\} = r_{ux}(k-l) \quad (24)$$

Making use of (24), (23) becomes

$$r_x(k) + \sum_{l=1}^{P} a_p(l) r_x(k-l) = \sum_{l=0}^{q} b_q(l) r_{ux}(k-l) \quad (25)$$

In its present form, the difference equation in (25) is not very useful. However by writing the cross-correlation $r_{ux}(k)$ in terms of autocorrelation $r_x(k)$ and the unit sample response of the filter, we may derive a set of equations relating the autocorrelation of $x(n)$ to the filter. This derivation can be found in [9].

The Yule Walker equations for the ARMA process in matrix form for any given channel tap is

$$R_H \underline{a}'' = \sigma_u^2 \underline{c} \quad (26)$$

where

$$R_H = \begin{bmatrix} r_h(0,l) & \cdots & r_h(-P,l) \\ \vdots & & \vdots \\ r_h(Q,l) & \ddots & r_h(Q-P,l) \\ \vdots & & \vdots \\ r_h(Q+P,l) & \cdots & r_h(Q,l) \end{bmatrix} \quad (27)$$

$$\underline{a}'' = [1 \quad a_l(1) \quad \cdots \quad a_l(P)]^T \quad (28)$$

$$\underline{c} = [c_l(0) \quad \cdots \quad c_l(Q) \quad 0 \quad \cdots \quad 0]^T \quad (29)$$

Note $\{c_l(q)\}_{q=0}^{Q}$ are the set of moving average (MA) coefficients. In practice the set of Yule Walker equations above can be solved by an optimization rule as will be shown in the following sections.

III. Proposed Receiver Based on the AR/ARMA Approaches

The proposed optimum receiver design is based on taking advantage of the AR model and the ARMA model. The two types of models are able to generate channel variates that contain both the Doppler information and the multipath information. Therefore, we can use the conventional rake receiver in [8] with each tap coefficient being generated by an AR/ARMA filter. This configuration is able to generate the channel without the need for channel estimation like in [11] and without the need for banks of correlators like in [12].

Figure 2 shows a block diagram of the proposed optimum receiver. In Figure 2, $\underline{u}(n)$ represents the white noise source inputs of the AR/ARMA generators. The rake receiver tap weights are determined by the channel responses that are provided by the AR/ARMA generators.

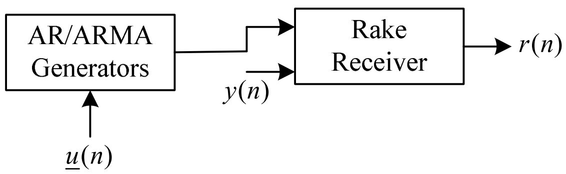

Figure 2: Proposed optimum receiver

IV. Computer Simulation

Figure 3 and Figure 4 show the autocorrelation functions of a PED-B channel model for different model orders, where a lower Doppler shift with a normalized value of 0.02 is used. It can be seen in these figures that the simulated autocorrelation functions are matched to a high degree with the theoretical autocorrelation ones, especially for the first 1000 lags.

The simulation procedure for evaluating the ARMA coefficients is the same for the AR ones, except that to find the values of the MA coefficients, we optimize them by using a mean square error (MSE) rule. The MSE is defined as:

$$MSE = \frac{|R_{th} - R_{sm}|^2}{n} \quad (30)$$

where R_{th} is the normalized theoretical autocorrelation function, R_{sm} is the normalized simulated autocorrelation function and n represents the number of simulation samples. The rule is that when the MSE reaches a certain threshold the simulation program stops and values for the AR coefficients and the MA coefficients are evaluated.

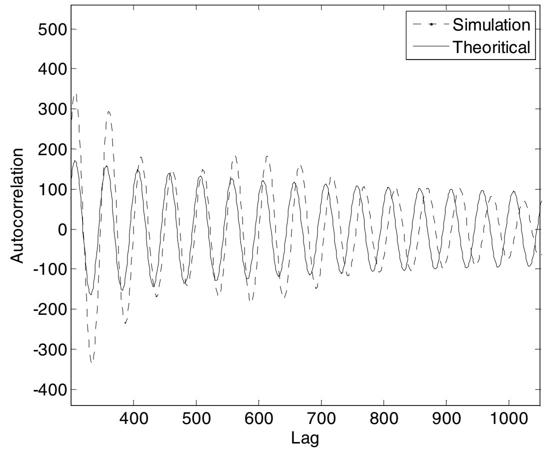

Figure 3: Autocorrelation of path 1 for a PED-B channel model, P=50: lags 300-1050

The benefit of using the ARMA model approach cannot be easily shown using figure comparisons. A measurement of the MSE is used instead. A comparison is illustrated in Figure 5 for 1500 samples. It is clear that increasing the degrees of freedom due to the ARMA approach produces smaller MSE values.

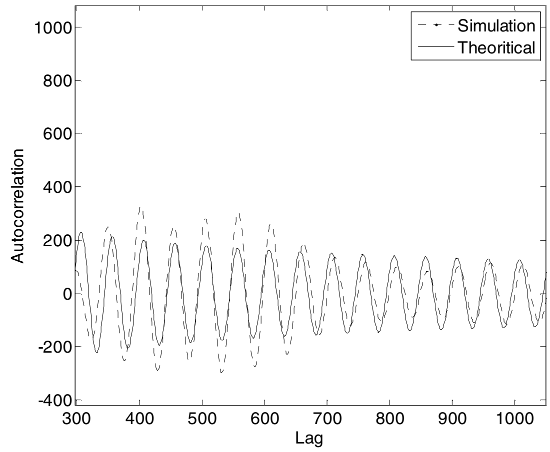

Figure 4: Autocorrelation of path 1 for a PED-B channel model, P=100: lags 300-1050

V. Performance Measurements

The power delay profile (PDP) shown in Figure 6 has been used in generating the performance curves in Figure 7 for the case of doubly selective fading, and when the normalized Doppler frequency shift is 0.05. The frequency selective slowly fading case has the power delay profile in Figure 6, with a normalized Doppler frequency shift of 0.001. It is clear that going from AR to ARMA with a lower nominator order (Q) gives better performance. Going from a lower to a denominator higher order P will improve the performance even further.

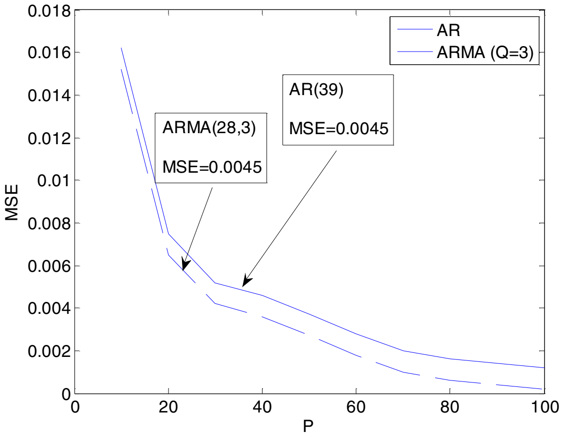

Figure 5: AR vs ARMA performance for different model orders

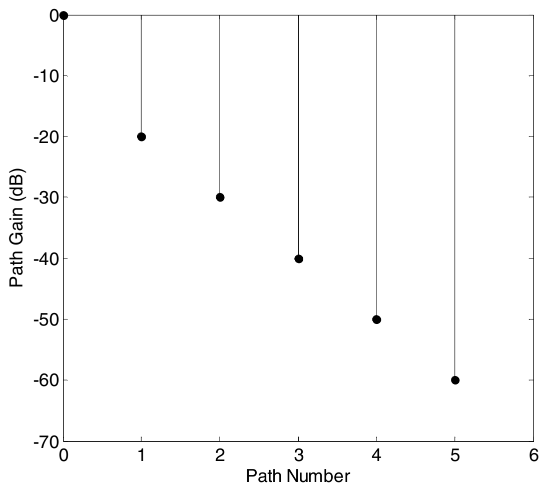

Figure 6: Power delay profile used in generating the performance curves in Figure 7

The frequency flat time selective fading case has one channel tap which has a gain of 0 dB and a normalized Doppler frequency shift of 0.05. In Figure 8, it can be shown that controlling the MSE using the ARMA approach will enhance the performance of the AR filter slightly when using the same model order P.

Figure 8 and Figure 9 show the performance of different model orders of both ARMA and AR models.

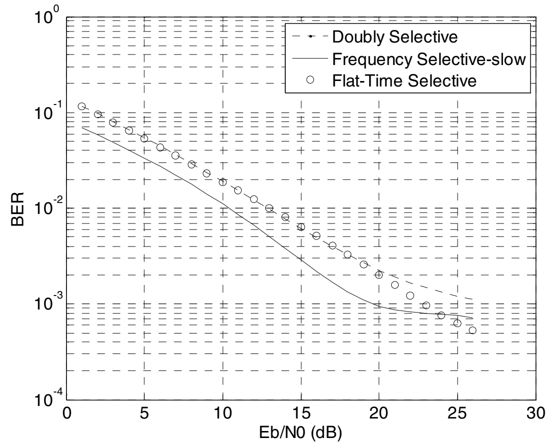

Figure 7: Performance of rake receiver under perfect channel knowledge

In Figure 10, a time selective case is shown where there is only one channel tap (flat fading). It is clear that less hostile environments show an increase in performance and even an enhancement in the AR or the ARMA modeling of the channel. It is assumed that the normalized Doppler frequency is 0.05 and the single channel tap has path gain of 0 dB.

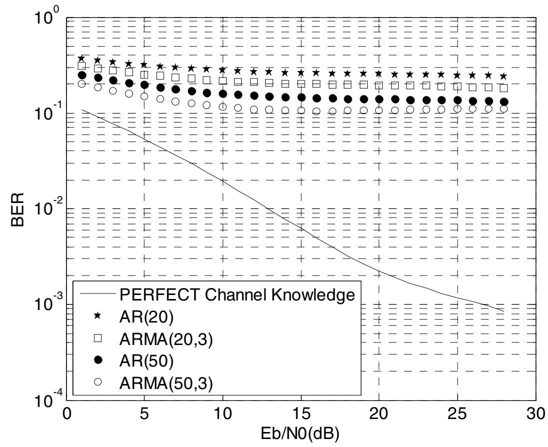

Figure 8: Performance of different orders of AR and ARMA models for a doubly selective fading channel

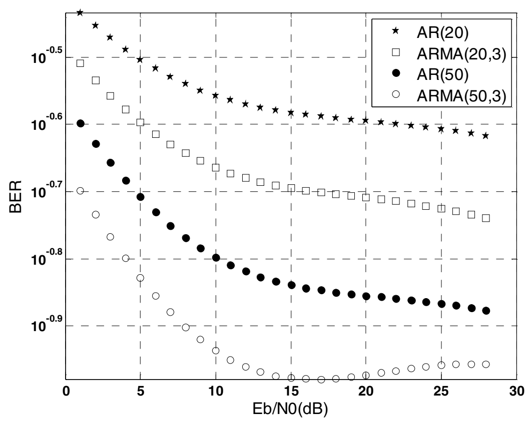

Figure 9: The effect of increasing the model order

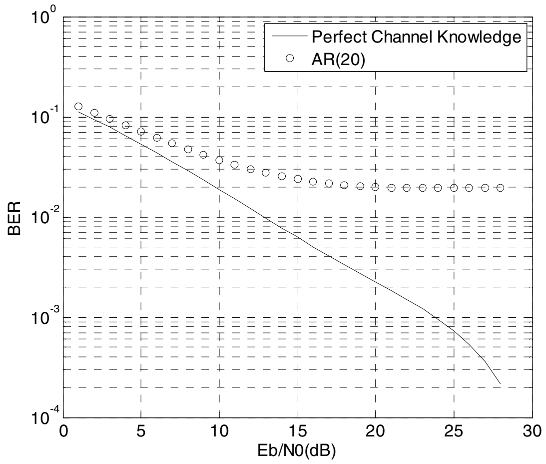

Figure 10: A time selective fading case with a normalized Doppler shift of 0.05

VI. Discussion And Conclusion

In this paper, simulation of doubly selective fading channels has been presented. We have studied the effects of AR and ARMA modeling for these channels. We have proposed a new type of optimum receiver that depends mainly on the AR and ARMA model parameters. The receiver has a less complex structure than the one in [11], with an acceptable level of performance. The performance of the proposed receiver improves with increasing the of AR/ARMA model order.

References

[1] B. Sklar, “Rayleigh fading channels in mobile digital communication systems Part I: Characterization,” IEEE Communications Magazine, Vol. 35, No, 7, pp. 90–100, July 1997

[2] J. Proakis, Digital Communications, McGraw-Hill, 1995

[3] W. Jakes, Microwave Mobile Communications, New Wiley, 1974

[4] D. J. Young and N. C. Beaulieu, “On the generation of correlated Rayleigh random variates by inverse discrete Fourier transform,” IEEE International Conference on Universal Personal Communications (ICUPC 1996)

[5] K. E. Baddour and N. C. Beaulieu, “Autoregressive Modeling for Fading Channel Simulation,” IEEE Transactions on Wireless Communications, Vol. 4, No. 4, pp. 1650-1662, July 2005

[6] M. F. Pop and N. C. Beaulieu, “Limitations of sum-of-sinusoids fading channel simulators,” *IEEE Transactions on Communications*, Vol. 49, No. 4, pp. 699–708, April 2001

[7] A. D. Abu Al-Khair and M. M. Banat**,** “Autoregressive stochastic modeling and tracking of doubly selective fading channels,” *IASTED International Conference on Communication Systems And Networks*, August 28-30, 2006, Palma de Mallorca, Spain.

[8] Proakis, J.G., Digital Communications, McGraw-Hill, 2000

[9] M. H. Hayes, Statistical Digital Signal Processing and Modeling, Wiley, 1996

[10] C. Loo and N. Secord, “Computer models for fading channels with applications to digital transmission”, IEEE Trans. Veh. Technol., vol. 40, no. 4, pp. 700–707, Nov. 1991

[11] Akbar M. Sayeed and Behnaam Aazhang “Joint Multipath-Doppler Diversity in Mobile Wireless Communications” IEEE Trans. Commun, vol. 47, no. 1, JANUARY 1999

[12] Fu Li, Heng Xiao and Jing Yang, “On Channel Estimation and Rake Receiver In a Mobile Multipath Fading Chann el” IEEE Trans. Commun, 1994