Using Pseudonoise Sequence Subcarrier Shuffling for Peak-to-Average Power Ratio Reduction in OFDM Systems

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Abstract–In this paper, we propose a new subcarrier assignment technique that is capable of substantially reducing the peak-to-average power ratio in orthogonal frequency division multiplexing wireless communication systems. The new technique is based on using pseudonoise sequence shuffling of the subcarrier assignments to data symbols within an orthogonal frequency division multiplexing block. Simulation results confirm the success of the proposed techniques in reducing the peak-to-average power ratio.

Keywords–Wireless Communications; orthogonal frequency division multiplexing; peak-to-average power ratio.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1][2] is an important multicarrier modulation technique for future high-speed wireless communication systems operating in the presence of multipath fading. OFDM applications include, among others, asymmetric digital subscriber line (ADSL) systems [1], wireless local area networks (LANs) [3] and digital audio and video broadcasting systems [4][5]. Additionally, OFDM is a strong candidate modulation scheme for future generations of wireless communication systems. In a high data rate system, where the channel is frequency selective, intersymbol interference (ISI) is likely to be present due to the multipath effect [6]. The higher the data rate, the more severe ISI is likely to be. With the aid of OFDM a high-rate data stream is split into several parallel lower-rate streams, that can be transmitted over the channel with reduced or no ISI [6]. Therefore, OFDM can help in avoiding the use of sophisticated channel equalizers that are, otherwise, needed to compensate for multipath-induced ISI.

In spite of the above-mentioned OFDM advantage in combating ISI, it does have some problems. By definition, an OFDM signal consists of the sum of a number of independently modulated subcarriers. This has the potential of producing a large peak-to-average power ratio (PAPR) when the subcarriers add up coherently. Large PAPR is a major disadvantage of OFDM signals. This is mainly because amplifiers with wide linear dynamic ranges become necessary as a result. Design of such amplifiers usually leads to increased complexity of the transmitter, and to reduced efficiency of the amplifiers. Without the use of such amplifiers a signal with large PAPR undergoes significant distortion, mainly due to saturation nonlinearities of the Mohammad M. Banat Jordan University of Science and Technology Department of Electrical Engineering PO Box 3030, Irbid 22110, Jordan Email: <u>m.banat@ieee.org</u>

amplifier. The immediate consequence of nonlinear amplifier distortion on a large-PAPR OFDM signal is usually reflected in the form of error rate degradation.

Several methods have been proposed to reduce PAPR in OFDM systems. Clipping the OFDM signal in the transmitter [7][8] is probably the simplest such method. By the use of clipping the transmitter forces signal values not to exceed a pre-specified threshold, such that the signal values remain within the amplifier linear range of operation. It has been shown in the literature that clipping can lead to bit error rate (BER) performance degradation [7], spectral efficiency reduction [9][10], and peak regrowth after digital to analog conversion, [7][11]. Oversampling the OFDM signal was shown in [12] to reduce the performance degradation due to clipping. Other PAPR reduction techniques include the tone reservation method [13][14], partial transmit sequences algorithm (PTS) [15][16][17], selected mapping algorithm (SLM) [18][19], pulse shaping [20], random phase updating [21], subblock phase weighting scheme [22], linear scaling [23], and precoding [24]. Most of these methods create correlation between the subcarriers. As a result, the reduction in PAPR achieved by these methods is relative and it requires added complexity and large processing overhead.

In this paper, we propose a new subcarrier assignment technique that is capable of reducing the PAPR. The technique we propose is to use pseudonoise code sequences to search for a shuffling of the default set of subcarriers that produces the smallest PAPR.

The remainder of this paper is organized as follows. In Section II, we define the PAPR of an OFDM signal, and discuss some related issues. In Section III, the proposed technique is explained. PAPR performance evaluations and comparisons are presented in Section IV based on computer simulations. In Section V, the performance of OFDM using the proposed techniques is presented. Section VI is devoted to discussions and conclusions.

II. SIGNAL AND PAPR MODELING

An OFDM signal to be transmitted during the time period $0 < t < T_s$ is defined by [25]:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n\Delta ft}$$
(1)

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where N is the OFDM symbol size (number of baseband modulated data symbols that constitute one OFDM symbol), X_n is the *n*-th data symbol (assumed to be complex-valued), and $n\Delta f$ is the *n*-th subcarrier frequency. Note that the subcarrier frequency separation is equal to $\Delta f = 1/T_s$. Let us define the data vector

$$\underline{X} = \begin{bmatrix} X_0 & X_1 & \cdots & X_{N-1} \end{bmatrix}^T$$
(2)

Using the definition of \underline{X} , (1) can be written in matrix form as follows:

$$x(t) = \frac{1}{\sqrt{N}} \underline{X}^T \underline{w}$$
(3)

where the transformation vector \underline{w} is defined as:

$$\underline{w} = \begin{bmatrix} e^{j0} \\ e^{j2\pi\Delta ft} \\ \vdots \\ e^{j2(N-1)\pi\Delta ft} \end{bmatrix}$$
(4)

The peak-to-average power ratio R of the signal x(t) is defined as the ratio of its peak instantaneous power and its average power. This can be mathematically expressed as [26]

$$R = \frac{\max_{0 < t < T_s} \left\{ |x(t)|^2 \right\}}{E \left\{ |x(t)|^2 \right\}}$$
(5)

Inspecting the signal expression in (1), it can be seen that a large PAPR value may be created through weighted summation of the N subcarrier sinusoids with different frequencies.

III. SUBCARRIER SHUFFLING METHOD FOR PAPR REDUCTION

In this section, we propose a new method for the reduction of PAPR. We call this method the method of shuffled subcarriers. This method is explained in the following subsection.

A. Method of Shuffled Subcarriers

In conventional OFDM, and as explained earlier, the *n*-th data symbol is assigned a subcarrier with frequency $n\Delta f$, where Δf is the subcarrier frequency separation and $n = 0, 1, \dots, N-1$. In the method of shuffled subcarriers a search is performed for shuffled sets of subcarriers that can

minimize the PAPR. This search operation is performed, for every OFDM symbol, just before the symbol-subcarrier assignment operation. The result is that the subcarriers are shuffled before they are assigned to the data symbols.

To illustrate the subcarrier shuffling procedure above with a simple example, let us consider an OFDM system with a block size N=32. Let the data symbol period be $T_s = 1$. The subcarrier frequency separation is, obviously, equal to unity. With no shuffling done, the default set of subcarriers is equal to $\{0,1,2,...,31\}$. If subcarrier shuffling is applied to find the shuffled set of subcarriers that minimizes the PAPR, we may end up, for example, with the set of subcarriers $\{3,12,25,...,5\}$. Note that the two sets of subcarriers above consist of the same subcarriers arranged in different orders. Note also that orthogonality between subcarriers is preserved; because the minimum separation between subcarriers is maintained. In addition, the bandwidth is not changed by using the proposed method, and no bandwidth expansion is incurred.

Comparing the default and shuffled sets of subcarriers in the example above reveals that the process of assigning subcarriers to data symbols is no longer a pure IFFT operation. The IFFT block has, therefore, to be replaced with another block. This also means that the FFT block in the receiver should be changed. The discrete Fourier transform can be replaced by what may be called the "generalized discrete Fourier transform" (GDFT). As a result of this, the transmitter should include an "inverse generalized Fourier transform" (IGDFT) block. If the IFFT operation in conventional OFDM is seen to be performed with the aid of the transformation vector w in (4), then the IGDFT operation can be performed with the aid of another transformation vector \underline{w}' . This new vector has the same rows of w rearranged in a shuffled order, i.e., w' can be computed through pre-multiplying w by a permutation matrix.

In the modified OFDM system the continuous-time signal x(t), originally given by (1), becomes

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi m(n)\Delta f t}$$
(6)

where it can be seen that *n* has been replaced by $m(n) \in \{0, 1, \dots, N-1\}$. Note that the sequence m(n) is a reordering of the values of the summation index *n*, and that it is specified by the minimum PAPR subcarrier shuffling obtained by the searching process.

B. System Description

In this sub-section, a description of the OFDM system with the proposed modifications introduced is provided. This has been depicted as in Figure 1 below. This figure shows an OFDM transmitter and receiver. An OFDM scheme with N orthogonal subcarriers is considered. Information bits are first fed into a frame generator and divided into N parallel bit streams by the serial-to-parallel (S/P) converter. The bit stream is mapped into a signal symbol sequence using one of the well-known modulation techniques. Parallel symbols are processed by an IGDFT block, as mentioned earlier. The order of subcarriers used in the IGDFT is one that minimizes the PAPR. This is done as explained above. After addition of the cyclic extension, the output signals are proceed by a parallel to serial (P/S) converter.



Figure 1: Modified OFDM System

At the receiver, the time domain signal is converted into parallel signals by an S/P converter. The cyclic extension is removed. The signal is then transformed using a GDFT so that it is ready for detection.

C. Using Pseudo-Noise (PN) Sequences in the Transmitter

There are two problems that have to be addressed in the proposed method above. The first is the search process that the transmitter performs to find the set of subcarriers that reduces the PAPR. The second problem is how the receiver computes the GDFT. This is because the specific GDFT depends on the set of subcarriers that the transmitter uses in the IGDFT and this set is not automatically known by the receiver.

To solve the problems above, a systematic procedure for the searching process is suggested. This procedure reduces the time needed by the transmitter to search for the new set of subcarriers. Additionally, this procedure provides a mechanism for the receiver to perform the GDFT. The proposed solution is to use pseudo-noise (PN) sequences in a contrived way. Only maximal length PN linear feedback shift register (SR) sequences will be used. The SR is assumed to consist of m stages, where

$$n = \log_2(N) \tag{7}$$

The details of the PN subcarrier shuffling procedure, with a special case of N = 8, m = 3, initial SR binary contents of (101) and feedback connections at the 1st and 3rd taps taken as an example, are as follows:

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- Generate a PN sequence of N−1 ones and zeros. For the special case above the sequence would be: 1010011.
- Generate an *m*-bit binary state, by taking the *m* leftmost bits of the PN sequence and follow this by a cyclic shift left of the sequence. Repeat this step N-1 times. For the special case above, this procedure would produce the following sequence of states: 101, 010, 100, 001, 011, 110, 111. Note that this sequence contains all the allowed states of binary contents of the SR, and that a change in the SR initial state implies only a cyclic shift of the sequence of states. As will be seen shortly, this simplifies the process of searching for the sequence of states that produces the least PAPR. Note also the absence of the SR.
- Use the symbol S_i, for i = 1,2,..., N-1 to represent the N-1 states in the generated sequence. For the special case above, the symbols are: S₁ = 101, S₂ = 010, ..., S₇ = 111. Note that S₁ is equal to the chosen initial state of the SR.
- Denote the decimal value of S_i by the symbol D_i . For our special case, $D_1 = 5$, $D_2 = 2$, ..., $D_7 = 7$.

- Generate an N×N permutation matrix P, such that for i=1,2,...,N-1 the *i*-th row is equal to <u>e</u>_{D_i}, the D_i-th column of the identity matrix of size N×N. Set the N-th column of P to <u>e</u>_N.
- By changing the initial condition of the PN shift register, a different permutation matrix can be generated. Except for the last row, which will always be equal to \underline{e}_N , different permutation matrices have different cyclic shifts of the same rows.
- The N-1 different permutation matrices can be applied to the data symbols for the purpose of finding the shuffling that produces the lowest possible PAPR. As mentioned above, the first row of the lowest PAPR permutation matrix uniquely specifies the needed SR initial state.
- The initial SR state that gives minimum PAPR should be known to the receiver, so that it is able to do GDFT. It is recommended that this information is transmitted separately prior to transmission of the current OFDM symbol. At the receiver the initial SR state information can be easily decoded, allowing the GDFT operation to be performed, and the OFDM symbol reception process to be completed.

IV. PAPR COMPARISONS

Computer simulations are used to evaluate the performance of the proposed PAPR reduction technique. Frequency flat (per subcarrier channel), slowly fading is assumed in the simulation. To measure the performance of the proposed technique, the PAPR complementary cumulative density function (CCDF) and the PAPR histogram are used. The CCDF displays the probability of the PAPR exceeding a certain value, while the histogram displays the statistical distribution of PAPR values. Ten thousand OFDM blocks were generated randomly to obtain the CCDFs.

Figure 2 shows PAPR CCDFs for N = 32 and N = 256 subcarriers per OFDM symbol using BPSK data modulation. Figure 3 shows PAPR CCDFs for N = 32 and N = 256 subcarriers per OFDM symbol using 16-QAM data modulation.

In both Figure 2 and Figure 3, PAPR CCDFs are shown for the default and the shuffled sets of subcarrier frequencies. It is clearly noted that the shuffled sets of subcarriers produce lower PAPR values than the default sets of subcarriers. Note that if a particular PAPR value, say $PAPR_o$, is selected, and the probability that the PAPR is greater than $PAPR_o$ is found for both methods, it can be easily seen that the modified sets of subcarriers produce lower probabilities than the default sets.



Figure 2: PAPR CCDF: BPSK Data Modulation, N = 32, 256



Figure 3: PAPR CCDF: 16-QAM Data Modulation, N = 32, 256

Figure 4 shows the PAPR histograms using BPSK data modulation and N = 32 for the default (solid line) and for the shuffled (dashed line) sets of subcarriers. Figure 5 shows the PAPR histograms using 16-QAM data modulation and N = 256 for the default (solid line) and for the shuffled (dashed line) sets of subcarriers.



Figure 4: PAPR Histogram: BPSK Data Modulation, N = 32

In the PAPR histograms of the default subcarrier sets in Figure 4 and Figure 5, it can be seen that large values occur with large probabilities. It can also be seen that the majority of PAPR values for the default sets falls in the region between 4 dB and 10 dB and concentrate with high probability between 5 dB and 7 dB. In the PAPR histograms of the shuffled subcarrier sets it can be seen that a majority of PAPR values falls in the region between 3 dB and 5 dB, and that large PAPR values occur with very small probabilities. It can also be noted that PAPR values concentrate with high probabilities between 3.5 dB and 4.5 dB.

In the PAPR histograms of the default subcarrier sets in Figure 4 and Figure 5, it can be seen that large values occur with large probabilities. It can also be seen that the majority of PAPR values for the default sets falls in the region between 4 dB and 10 dB and concentrate with high probability between 5 dB and 7 dB. In the PAPR histograms of the shuffled subcarrier sets it can be seen that a majority of PAPR values falls in the region between 3 dB and 5 dB, and that large PAPR values occur with very small probabilities. It can also be noted that PAPR values concentrate with high probabilities between 3.5 dB and 4.5 dB. This behavior is due to the fact that the PAPR has been substantially reduced by the use of the proposed subcarrier shuffling technique.



Figure 5: PAPR Histogram: 16-QAM Data Modulation, N = 256

V. ERROR PROBABILITY PERFORMANCE OF THE MODIFIED SYSTEM

In this section, simulation is used to compute the error probability for modified OFDM systems. These results are also compared with conventional OFDM systems. The effect of the power amplifier is taken into account during simulation of both modified and conventional OFDM systems. So that our focus remains on assessing the effectiveness of subcarrier shuffling in improving OFDM performance, the simulations below are limited to systems operating in frequency flat slowly fading channels.

In practical OFDM systems, the signals are amplified by a power amplifier which is peak-power limited. If the envelope of the transmitted signal is larger than the PA's saturation point at any time instant, the signal at this instant will be clipped creating nonlinear distortion and consequentially increasing the probability of error.

As a first test, two OFDM systems are simulated, a conventional one and a modified one. N = 32 subcarriers per OFDM symbol is assumed throughout, with BPSK data modulation for the two systems. A slowly fading Rayleigh wireless channel is assumed. The output backoff (OBO) is defined as:

$$OBO = 10\log_{10}\left(\frac{P_{\max,out}}{P_{out}}\right) \tag{8}$$

where $P_{\max,out}$ is the maximum output power of a power amplifier (saturation power) and P_{out} is the mean power of the power amplifier output signal. The PA is assumed to enter the saturation region when the output backoff equals 4 dB.

Figure 6 shows the BER versus the average signal to noise ratio (SNR) per bit. The first case plotted in Figure 6 (called ideal OFDM) the PA is ideal, meaning that no signal clipping occurs (OBO is infinity). The second case plotted in the same figure represents conventional OFDM. In this case, the effect of the PA is included, meaning that the PAPR problem exists. It is clear from the figure that large degradation in the BER performance is suffered, a accompanied by an error floor that takes place as a result of the nonlinearity of the PA.

The BER of the modified OFDM system is also shown in Figure 6. It is noted that an improvement in the BER performance is achieved. However, the performance is still away from the required performance that could be obtained using ideal PA. Our assumed 4 dB value of the OBO is relatively small, and practical PAs have *OBO* values that can reach 9dB. Therefore, the simulation above has been repeated assuming equals 6 dB.



Figure 6: BER of BPSK over a Rayleigh channel versus the average SNR per bit for different OFDM systems, N=32, OBO = 4 dB

Figure 7 shows BER versus average SNR per bit taking into consideration the new value of the *OBO*. From this

figure, it can be seen that conventional OFDM gives unacceptable BER performance compared to ideal OFDM. Modified OFDM introduces a large improvement in the BER performance, approacing performance of ideal OFDM. It can be seen in Figure 7 that the BER degrades at high bit SNRs; because it is not possible to reduce all peaks below the PA point of saturation.

To further test the proposed method it is applied it to a 32-subcarrier 16-QAM OFDM system in slow Rayleigh fading. An OBO value of 8 dB is assumed. Figure 8 shows the symbol error rate (SER) performance. This figure demonstrates performance improvements through the proposed modification similar to those obtained with BPSK above. As can be seen in Figure 8, modified OFDM results in substantial performance improvement compared to conventional OFDM system, and its SER performance comes close to that of ideal OFDM.



Figure 7: BER of BPSK over a Rayleigh channel versus the average SNR per bit for different OFDM systems, N=32, OBO = 6 dB



Figure 8: SER of 16-QAM over a Rayleigh channel versus the average SNR per symbol for different OFDM systems. OBO = 8 dB

VI. CONCLUSION AND FUTURE WORK

In this paper, a pseudonoise sequence-based subcarrier shuffling technique has been proposed to reduce PAPR in OFDM systems. The concept is based on shuffling the set of subcarriers before assigning them to the data symbols; in order to obtain a subcarrier assignment that is able to reduce the PAPR. The idea of shuffling is to decrease the possibility of peaks of the subcarriers signals occurring at the same time. Through a set of computer simulations, the proposed technique has been found to be effective in reducing the PAPR and improving the system error rate performance.

The most important possible future extension of this work would be to design more efficient searching algorithms for low-PAPR shuffled subcarrier sets. Another possible extension would be to study the effects of using only a subset of positive integers smaller than a given power of 2 to generate the subcarrier frequencies.

References

- S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," IEEE Transactions on Communications Technology, Vol. 19, No. 5, pp. 628–634, October 1971
- [2] J. A. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," IEEE Communications Magazine, Vol. 28, No. 5, pp. 5–14, May 1990
- [3] R. van Nee, G. Awater, M. Morikura, H. Takanashi, M. Webster, and K.W. Halford, "New high-rate wireless LAN standards," IEEE Communications Magazine, Vol. 37, No. 12, pp. 82–88, December 1999
- [4] ETSI ETS 300 401, "Radio broadcasting systems; digital audio broadcasting (DAB) to mobile, portable and fixed receivers," ETSI, Technical Report, February 1995
- [5] ETSI ETS 300 744, "Digital video broadcasting (DVB); frame structure, channel coding and modulation for digital terrestrial television (DVB-T)," ETSI, Technical Report, March 1997
- [6] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," IEEE Communications Magazine, Vol. 33, No. 2, pp. 100–109, February 1995
- [7] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," IEEE Vehicular Technology Conference, May 1997, pp. 1634-1638
- [8] U. K. Kwon, D. Kim, K. Kim, and G. H. Im, "Amplitude clipping and iterative reconstruction of STBC/SFBC-OFDM signals," IEEE Signal Processing Letters, Vol. 14, No. 11, pp. 808-811, November 2007
- [9] R. O'Neill and L. N. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals," IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, September 1995, pp. 71-75
- [10] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," IEEE Wireless Communication, Vol. 12, No. 2, pp. 56-65, April 2005
- [11] H. Ochiai and H. Imai, "On clipping for peak power reduction of OFDM signals," IEEE Global Telecommunications Conference, May 2000, pp. 731-735
- [12] M. Sharif and B. H. Khalaj, "Peak to mean envelope power ratio of oversampled OFDM signals: An analytical

approach," IEEE International Conference on Communications, June 2001, pp. 1476-1480

- [13] L. J. Cimini and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," IEEE Communications Letters, Vol. 4, No. 3, pp. 86-88, March 2000
- [14] S. H. Han and J. H. Lee, "PAPR reduction of OFDM signals using a reduced complexity PTS technique," IEEE Signal Processing Letters, Vol. 11, No. 11, pp. 887-890, November 2004
- [15] J. Tellado, Multicarrier Modulation With Low PAR: Applications To DSL And Wireless, Kluwer International Series In Engineering And Computer Science, Kluwer Academic Publishers, September 2000
- [16] A. D. S. Jayalath and C. Tellambura, "Adaptive PTS approach for reduction of peak-to-average power ratio of OFDM signal," Electronics Letters, Vol. 36, No. 14, pp. 1226-1228, July 2000
- [17] A. D. S. Jayalath and C. Tellambura, "SLM and PTS peakpower reduction of OFDM signals without side information," IEEE Transactions on Wireless Communications, Vol. 4, No. 5, pp. 2006-2013, September 2005
- [18] A. D. S. Jayalath, C. Tellambura, and H. A. Wu, "Reduced complexity PTS and new phase sequences for SLM to reduce PAP of an OFDM signal," IEEE Vehicular Technology Conference, May 2000, pp. 1914-1917
- [19] N. V. Irukulapati, V. K. Chakka, and A. Jain, "SLM based PAPR reduction of OFDM signal using new phase sequence," Electronics Letters, Vol. 45, No. 24, pp. 1231-1232, November 2009
- [20] S. B. Slimane, "Peak-to-average power ratio reduction of OFDM signals using pulse shaping," IEEE Global Telecommunications Conference, April 2000, pp. 1412-1416
- [21] H. Nikookar and K. S. Lidsheim, "PAPR reduction of OFDM by random phase updating," IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, September 2002, pp. 814-818
- [22] H. G. Ryu and K. J. Youn, "A new PAPR reduction scheme: SPW (subblock phase weighting)," IEEE Transactions On Consumer Electronics, Vol. 48, No. 1, pp. 81-89, February 2002
- [23] H. Ochiai, "Performance analysis of peak power and bandlimited OFDM system with linear scaling," IEEE Transactions on Wireless Communications, Vol. 2, No, 5, pp. 1055-1065, September 2003
- [24] S. B. Slimane, "Reducing the peak-to-average power ratio of ofdm signals through precoding," IEEE Transactions On Vehicular Technology, Vol. 56, No. 2, pp. 686-695, March 2007
- [25] P. H. Moose, "A Technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction," IEEE Transactions on Communications, Vol. 42, No. 10, pp. 2908-2914, October 1994
- [26] K. Yang and S. I. Chang, "Peak-to-Average Power Control in OFDM Using Standard Arrays of Linear Block Codes," IEEE Communications Letters, Vol. 7, No. 4, pp. 174-176, April 2003