# AUTOREGRESSIVE STOCHASTIC MODELING AND TRACKING OF DOUBLY SELECTIVE FADING CHANNELS

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**ABSTRACT**

This paper presents a new autoregressive (AR) stochastic modeling and tracking method of doubly selective fading channels. The AR model is used to generate a wide-sense stationary uncorrelated scattering (WSSUS) channel impulse response. A Kalman-based tracking algorithm is then designed to continuously track the channel at the receiver.

**KEY WORDS**

Frequency selective fading, time selective fading, AR modeling, Kalman filter, channel tracking.

## 1. Introduction

A wireless communication channel is said to be doubly selective when it exhibits both time- and frequency-selective fading. Such channels are encountered in many current applications where communication takes place over wide bandwidths, at high carrier frequencies, and under high mobility conditions [1].
In a typical wideband application, frequency selectivity is likely to cause intersymbol interference (ISI); because the transmitted signal bandwidth exceeds the channel coherence bandwidth [2]. The presence of ISI usually requires the use of equalization at the receiver [2]. Time selectivity is usually the result of high mobility of the communicating device. It manifests itself in the form of fast channel time variations [1]. Therefore, a doubly selective fading channel is a rapidly time-variant one. Therefore, in addition to the need for an equalizer to take care of ISI, a fast and accurate tracking algorithm has to be implemented to handle the channel time variation [1].
Frequency selective fading communication channels are generally described using tapped delay line models [2]. Contrary to situations that may arise in other applications, the channel tracking algorithm, mentioned above, has to follow the actual values of the delay line tap coefficients rather than their average values.
A popular approach to incorporate time variations into the delay line model is to represent the tap coefficients using deterministic time varying basis expansion fucntions (e.g., using complex exponentials) [3], [4]. This sort of modeling is particularly useful when the multipath is mainly caused by a few strong reflectors and when path delays exhibit variations due to the kinematics of the mobiles [3], [4], [5].
A more general approach to describe time-varying communication channels is by treating the delay line coefficients as lowpass Gaussian uncorrelated stationary random processes [1], [2], [6]. This approach is suitable for situations where large a number of scatterers exist.
Our channel modeling and tracking algorithm in this paper is based on the more general statistical approach to describe the channel tap coefficients. The time evolution of each tap coefficient will be modeled as an autoregressive (AR) random process. AR modeling of time varying channel tap coefficients was first proposed in [7].
Several algorithms have been proposed for the generation of Rayleigh random variates [8], [9], [10], [11] that can be used to represent the envelopes of the tap coefficients. These algorithms can be generally classified to follow either the sum-of-sinusoids (SOS) approach or the inverse Discrete Fourier Transform (IDFT) approach [12]. It was shown in [13] that the classical Jakes' simulator (which is an SOS-based approach) produces fading signals that are not wide sense stationary. On the other hand, IDFT-based techniques, are known to be storage-demanding, even though accurate [12], [14], [15]. It was shown in [1] that the AR model can efficiently simulate a WSSUS fading channel with an accuracy close to that of the IDFT model and with a much less storage requirements.

## 2. Fading Channel Modeling

Consider the following multipath fading channel discrete-time input/output relation [16]

$$y(n) = \sum_{l=1}^{L-1} h(n,l)\,x(n-l) + \xi(n) \qquad (1)$$

where $x(n)$ and $y(n)$ are the channel input and output, respectively, $h(n,l)$ is the channel impulse response as function of time $n$ and delay $l$, and $\xi(n)$ is a zero-mean additive white Gaussian noise (AWGN) process with variance $\sigma_\xi^2$. We are assuming a WSSUS channel. In the sequel, we will use the AR model to generate the channel taps $\{h(n,l)\}$, for $l = 0,1,\cdots,L-1$ and $n = 0,1,\cdots$.

Modeling the channel impulse response as an AR process means that $h(n,l)$ depends on a subset of its previous values $h(n-1,l)$, $h(n-2,l)$, $\cdots, h(n-P,l)$ as follows [17]

$$h(n,l) = \sum_{p=1}^{P} a_l(p) \cdot h(n-p,l) + u_l(n) \tag{2}$$

where $P$ is the AR model order, $\underline{a_l} = \left[a_l(1)\, a_l(2) \cdots a_l(P)\right]^T$ is the channel tap coefficient vector, and $u_l(n)$ is a zero-mean complex valued white Gaussian stochastic process, with variance $\sigma_u^2$, that is uncorrelated with $\xi(n)$ and that has the autocorrelation function given by

$$\begin{aligned} r_u(k) &= \mathrm{E}\left[u(n)u^*(n-k)\right] \\ &= \sigma_u^2 \cdot \delta(k) \end{aligned} \tag{3}$$

This channel model was used in [1] with a wide sense stationary correlated scattering (WSSCS) channel, where each tap-vector was a function of the previous $P$ tap vectors. Even though we are using a simplified model by assuming uncorrelated scattering, it will be seen later that this model allows us to efficiently track the channel impulse response time variations.

Our first objective is to determine a mean-squared-error (MSE) optimized estimate of the coefficient vector $\underline{a_l}$.

Multiplying (2) by $h^*(n-k,l)$ on both sides and taking the expected value yields for $k = 0,1,2,\ldots,P$ [17]

$$r_h(k,l) = \sum_{p=1}^{P} a_l(p) \cdot r_h(k-p) + \sigma_u^2 \delta(k) \tag{4}$$

where use has been made of the channel autocorrelation function defined as

$$r_h(k,l) = \mathrm{E}\left[h(n,l)h^*(n-k,l)\right] \tag{5}$$

Next, we note that for any $k > 0$, (4) can be written in matrix form as follows

$$R_h(l) \cdot \underline{a_l} = \underline{r_h}(l) \tag{6}$$

where

$$R_h(l) = \begin{bmatrix} r_h(0,l) & r_h^*(1,l) & \cdots & r_h^*(P-1,l) \\ r_h(1,l) & r_h(0,l) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ r_h(P-1,l) & \cdots & \cdots & r_h(0,l) \end{bmatrix} \tag{7}$$

and

$$\underline{r_h}(l) = [r_h(1,l)\, r_h(2,l) \cdots r_h(P,l)]^T \tag{8}$$

In order to solve for the unknown coefficient vector $\underline{a_l}$, we have to calculate the matrix $R_h(l)$ and the vector $\underline{r_h}(l)$. In [1] this is done through tracking the elements of $R_h(l)$ and $\underline{r_h}(l)$ using a recursive least squares (RLS) algorithm, or using a closed form approximation based on higher order statistics to give an estimate of $r_h(k,l)$ once per data block of size $N$. This procedure has a major drawback regarding the first step. The autocorrelation of the fading channel is very difficult to track using an adaptive algorithm. This is because the channel autocorrelation over a window is usually varying faster than the convergence time of these algorithms. As will be explained later in the paper, we will analytically calculate $\underline{a_l}$ using the Bessel function channel correlation model.

## 3. Channel Tracking

Consider the channel tap coefficient vector

$$\underline{h}(n) = [h(n,0)\, h(n,1) \cdots h(n,L-1)]^T \tag{9}$$

which can be alternatively written in the form

$$\underline{h}(n) = \sum_{p=1}^{P} A(p) \cdot \underline{h}(n-p) + \underline{u}(n) \tag{10}$$

where $\underline{u}(n)$ is an $L$-element vector whose values are taken from a complex zero-mean white Gaussian stochastic process with variance $\sigma_u^2$ [1], i.e.,

$$R_u = \sigma_u^2 \cdot I_{L\times L} \cdot \delta(k) \tag{11}$$

$A(p)$ is the diagonal matrix given by:

$$A(p) = \mathbf{diag}\left\{a_1(p), a_2(p), \cdots, a_L(p)\right\} \tag{12}$$

Note that $a_l(p)$ corresponds to the $p$th coefficient associated with the $l$th tap. Let's now define the augmented channel vector

$$\hat{\underline{h}}(n) = [\underline{h}^T(n)\, \underline{h}^T(n-1) \cdots \underline{h}^T(n-P+1)]^T \tag{13}$$

Let's also make the following two definitions

$$\hat{A} = \begin{bmatrix} A(1) & A(2) & \cdots & A(P) \\ I_L & O_L & \cdots & O_L \\ \vdots & \ddots & \ddots & \vdots \\ O_L & \cdots & I_L & O_L \end{bmatrix} \quad (14)$$

$$J = [I_{L\times L} \underbrace{O_L \cdots O_L}_{P-1}]^T \quad (15)$$

where $I_L$ is an $L \times L$ identity matrix and $O_L$ is an $L \times L$ matrix of zeros. Using the new definitions above we can rewrite (10) in the form [1]

$$\hat{\underline{h}}(n) = \hat{A} \cdot \underline{h}(n-1) + J \cdot \underline{u}(n) \quad (16)$$

Now let's define the transmitted vector

$$\underline{x}(n) = [x(n)\ x(n-1) \cdots x(n-L+1)]^T \quad (17)$$

This enables rewriting (1) as follows

$$y(n) = \left(J\underline{x}(n)\right)^T \hat{\underline{h}}(n) + \xi(n) \quad (18)$$

Now, with the aid of (16) and (18), we can use a Discrete Kalman filter to track the channel vector $\hat{\underline{h}}(n)$. In the following two subsections present two tracking algorithms.

### 3.1. *Tracing Algorithm-1*

- Initialization:

Initialize the channel tap coefficient vector to $\hat{\underline{h}}(0/0) = \underline{\mathbf{0}}$, where $\underline{\mathbf{0}}$ denotes a vector of $L$ zeros.

Define the initial matrix $Q(0/0) = O_{PL}$, where $O_{PL}$ is a $PL \times PL$ matrix of zeros.

- Recursion:

for $n = 1, 2, \cdots$ compute

$$\hat{\underline{h}}(n/n-1) = \hat{A}\hat{\underline{h}}(n-1/n-1)$$

$$Q(n/n-1) = \hat{A}Q(n-1/n-1)\hat{A}^H + \sigma_u^2 JJ^H$$

$$\underline{K}(n) = Q(n/n-1)J\underline{x}^*(n) \cdot \left[\underline{x}^T(n)J^T Q(n/n-1)J\underline{x}^*(n) + \sigma_\xi^2\right]^{-1}$$

$$\hat{\underline{h}}(n/n) = \hat{\underline{h}}(n/n-1) + \underline{K}(n)\left[y(n) - \underline{x}^T(n)J^T\hat{\underline{h}}(n/n-1)\right]$$

$$Q(n/n) = \left[I_{PL} - \underline{K}(n)\underline{x}^T(n)J^T\right]Q(n/n-1)$$

The matrices $Q(n/n)$, $Q(n/n-1)$, and $\hat{A}$ are all $LP \times LP$ square matrices, while $\underline{K}(n)$ is an $LP \times 1$ gain vector [1], [17].

Note that this algorithm allows the calculation of the matrix $Q(n/n)$ offline. The resulting values can then be used when the recursion is run. Hence the amount of computations required can be reduced at the cost of some storage requirements.

### 3.2. *Tracing Algorithm-2*

- Initialization:

$\hat{\underline{h}}(1/0) = \underline{\mathbf{0}}$

$E(1/0) = O_{PL}$, where $O_{PL}$ is a $PL \times PL$ matrix of zeros.

- Recursion:

for $n = 1, 2, \cdots$ compute

$$r(n) = \left[\underline{x}^T(n)J^T E(n/n-1)J\underline{x}^*(n) + \sigma_\xi^2\right]^{-1}$$

$$K(n) = \hat{A}E(n/n-1)J\underline{x}^*(n)r(n)$$

$$\hat{\underline{h}}(n+1/n) = \left[\hat{A} - K(n)\underline{x}(n)^T J^T\right] \cdot \hat{\underline{h}}(n/n-1) + K(n)y(n)$$

$$E(n+1/n) = \hat{A}E(n/n-1) \cdot \left[I - J\underline{x}^*(n)r(n)\underline{x}^T(n)J^T E(n/n-1)\right] \cdot \hat{A}^H + \sigma_u^2 JJ^H$$

U nlike the first algorithm, this algorithm does not involve any quantities that can be calculated offline. However, it requires fewer computations than the first algorithm.

## 4. SIMULATION

We have used two methods to generate the channel. In the first method we used the AR model suggested in [14], while in the second one we used the Jakes model given in [18, Chapter 3]. The stationarity of the channel generated using the AR model was achieved, as in [12], through a start-up procedure that is based on the Levinson recursion. With the Jakes simulator we have used a total of 200 sinusoids that were added at each time instant.

Since the channel impulse response is modeled as an AR process, the receiver should know $\hat{A}$, the coefficient matrices of the AR model, before running the Kalman channel tracking algorithm. These coefficients will be determined using analytical calculation. Assuming a symbol period $T$ and a Doppler spread $f_d$, we use the Bessel function channel correlation model given by

$$r_h(k,l) = J_0(2\pi f_d kT) \quad (19)$$

to solve the Yule-Walker equations given in (6). In doing so, we can use the Levinson-Durbin recursion to calculate the channel tap coefficient vector $\underline{a_l}$ and the variance of the white process $u(n)$.

We will study the performance of the tracking algorithm in three cases. These cases differ in the amount of information available at the receiver. The receiver tracks the channel with the aid of the AR model. In doing so, the type of channel correlation and the order of the model are parameters that shall be specified at the receiver at the beginning.

To study the functionality of the proposed tracking procedure, we will gradually decrease the amount of information available to the receiver. We have limited our simulations to tracking algorithm-1; because algorithm-2 gives very similar results.

First we will assume that the receiver knows the type of channel correlation and the order used in generating the channel using the AR model. Based on this knowledge, the receiver uses a model similar to the one used to generate the channel in tracking it. Part of the results of this experiment are shown in **Figure 1** and **Figure 2** (the remaining results are similar and have been omitted to minimize the paper size). The performance of the tracking algorithm over the first 500 training symbols is shown. In this example, we have generated a 4-tap, WSSUS fading channel with a normalized Doppler spread of $f_d T = 0.03$, and an exponential power delay profile, given by $\phi(lT) = 2^{-l}$. The order of the AR model used to generate this channel is 50. As one may expect, the tracking algorithm in this case gives almost perfect match to the actual channel for all taps.

Next we relax the requirement that the receiver knows the order of the channel model. Hence we arbitrarily select a model order and use it in the receiver. Part of the results of this case are shown in Figure 3 and Figure 4. We have generated a 4-tap channel using a AR(100) model with the same power delay profile and normalized Doppler spread as in the first case. At the receiver side, we have arbitrarily selected a AR(50) model for tracking. Once again the performance of the tracking algorithm for the first 500 training symbols is shown. As we can see, the algorithm is still capable of giving fair results even with order mismatches.

Finally, we have used the Jakes Sum of Sinusoids channel simulator to generate a channel with the same power delay profile and normalized Doppler spread as before. In this case, the receiver must arbitrarily select an order for the AR model to use it in the tracking. An AR(20) model was selected. The results are shown in Figure 5 and Figure 6. Once again the algorithm has proven its functionality even though the channel is generated using a model other than the AR model.

Though the tracking performance had suffered some degradation from the first to the third experiments, the level of performance that the algorithm reaches is reasonable for the purpose of channel equalization. This can be understood once we note that the degradation is mainly in the later taps while the former taps still enjoy good tracking. Since the contribution of the former taps in the ISI is larger than the later taps, then we conclude that the tracking results are reasonable and are quite good to be used for equalization.

## 5. Conclusions

We have investigated the potentials of the autoregressive stochastic model in modeling the doubly selective fading channel. It was shown that we can use the Bessel function channel correlation to solve for the model coefficients; and thus to generate the WSSUS channel. After that we have proposed two tracking algorithms both based on the discrete Kalman filtering to track the fading channel online. It was shown that the tracking algorithms have the capability to track a variety of fading channels once the symbol duration and the channel Doppler spread are known. The order of the model used in the tracking algorithms represents a fine control for the accuracy of the process, but at the cost an increase in computational complexity.

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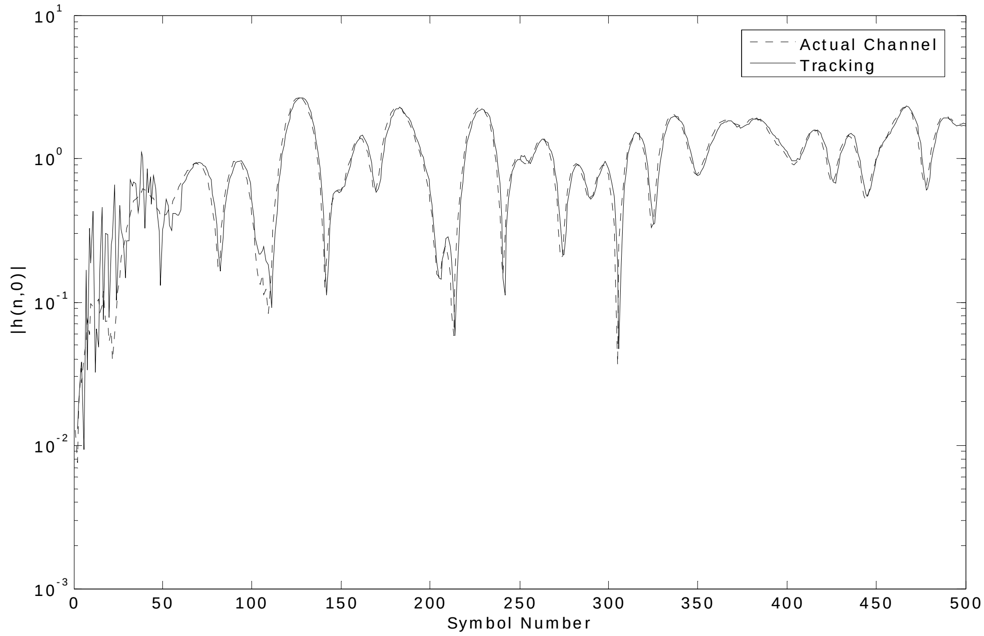


Figure 1: Channel tracking of $h(n,0)$ using algorithm-1, $f_d T = 0.03$ and an AR model order of 50

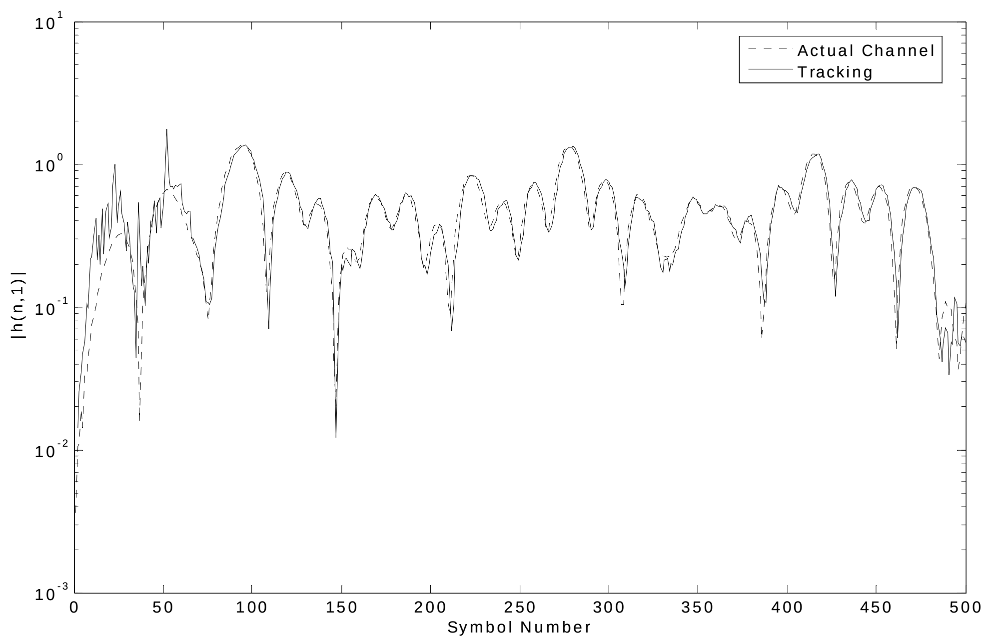


Figure 2: Channel tracking of $h(n,1)$ using algorithm-1, $f_d T = 0.03$ and an AR model order of 50

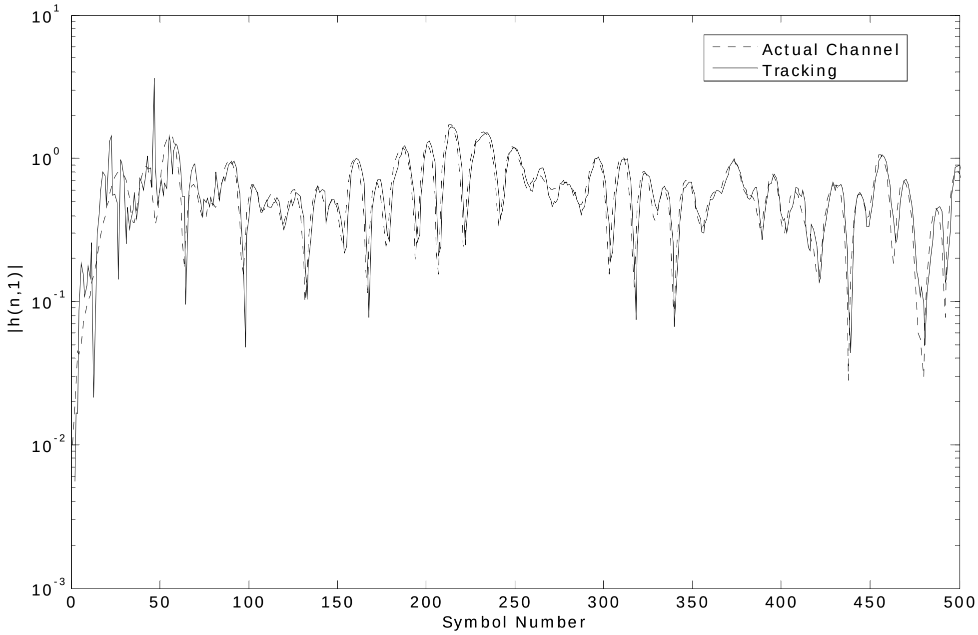


Figure 3: Channel tracking of $\boldsymbol{h}(\boldsymbol{n},1)$ with order mismatch: channel generated using AR(100) and tracked using AR(50)

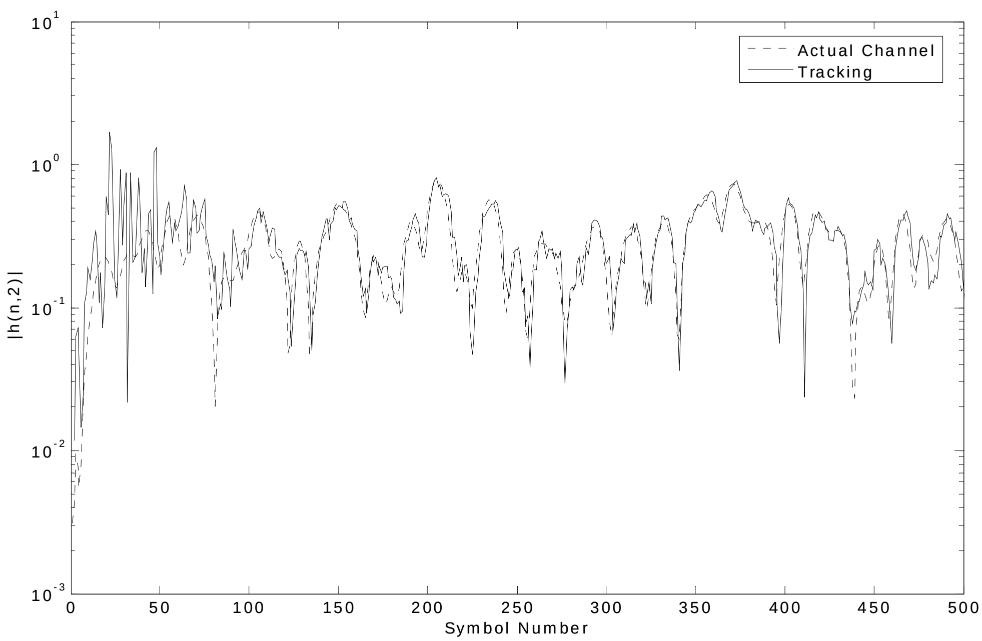


Figure 4: Channel tracking of $\boldsymbol{h}(\boldsymbol{n},2)$ with order mismatch: channel generated using AR(100) and tracked using AR(50)

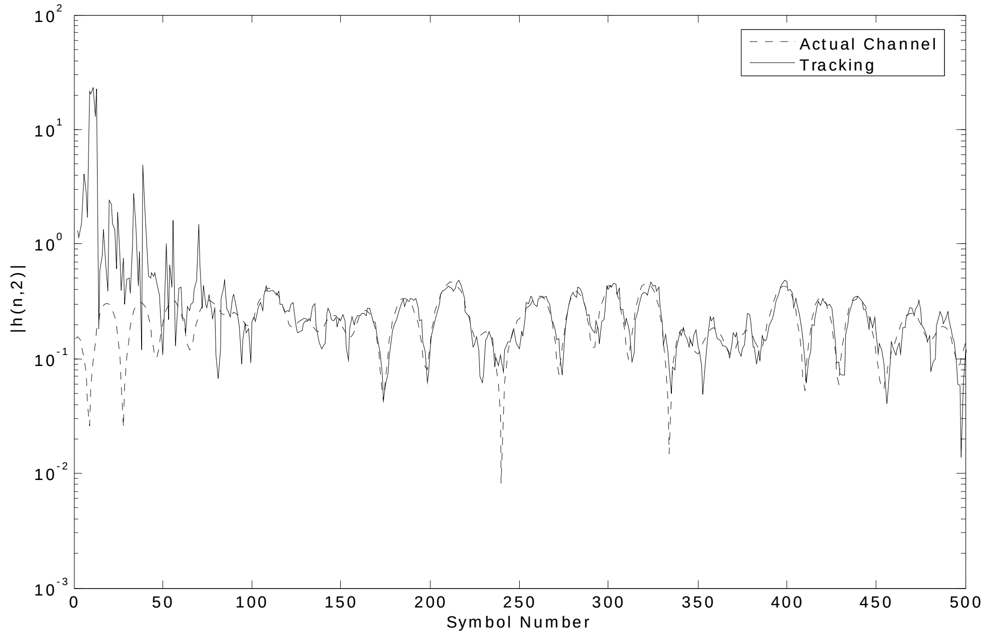


Figure 5: Tracking $\boldsymbol{h}(\boldsymbol{n},2)$ of a channel generated using Jakes model with M=200 using an AR(50) model

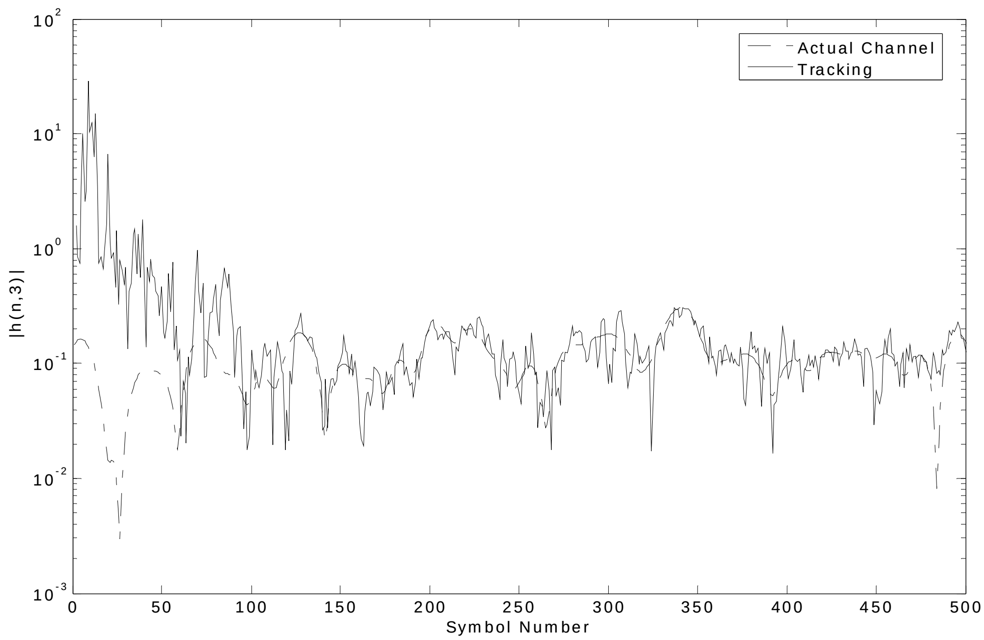


Figure 6: Tracking $\boldsymbol{h}(\boldsymbol{n},3)$ of a channel generated using Jakes model with M=200 using an AR(50) model