

Bit Error Rate of Majority Vote Combining Symbol Slicing Noncoherent Binary Optical Heterodyne FSK Receivers

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Summary

This paper investigates the performance of binary optical heterodyne FSK when a symbol slicing majority vote combining receiver is used. In particular, emphasis will be placed on the role of the proposed receiver structure in reducing the performance loss due to laser phase noise. For the sake of a simple receiver configuration, noncoherent demodulation of FSK is utilized. Bit error rate (BER) results obtained in this work demonstrate the effectiveness of the proposed reception mechanism in countering the effects of laser phase noise.

1 Introduction

Performance of heterodyne optical receivers affected by semiconductor laser phase noise and photodetector shot noise has been the subject of intensive research over the past two decades [1–17]. It is demonstrated in [1] that an extremely small value of the product of the phase noise linewidth and the signaling interval (linewidth-duration product) is required so that a PSK system can operate satisfactorily. In [2] it is stated that a narrow laser linewidth is a requirement for practical realization of heterodyne systems. A heterodyne DPSK system is studied in the presence of phase noise in [3] where the error probability is expressed in an infinite series form. The error probability of a DPSK system affected by phase noise is computed in [4] using numerical integration. An error rate floor due to laser phase noise is demonstrated in [5] and [6] for heterodyne ASK and FSK systems. In [7] and [8] the performance of several heterodyne systems is analyzed in the presence of phase noise. Heterodyne ASK and FSK receivers are studied in [9] where the intermediate frequency is treated as a stochastic process. Linewidth requirements in heterodyne optical receivers are studied in [10] and [11]. Laser linewidth effects on the performance of ASK and FSK systems are studied in [12]. [13] presents a study of on-off keying (OOK) and FSK systems using postdetection filtering and a Brownian motion model of phase noise. The same approach to phase noise modeling is also used in [14] and [15]. DPSK receivers using weighting to counter phase noise are analyzed in [16].

Most published literature assumes receiver models that are optimum in the absence of laser phase noise [18].

Very little progress has been achieved in optimizing receiver design taking phase noise into account [13, 17, 19–22]. This is primarily due to the difficulty in statistically characterizing phase noise random variables in the receiver. Receiver decision variables including phase noise contributions have been studied by many authors so far [23–32].

In [23], for instance, analytical expressions for the probability density function (PDF) and the moment generating function (MGF) of a filtered phase noise random variable are derived using a small phase noise approximation. Studies of phase noise are presented in [24] and [25] based on the evaluation of moments. A substantial amount of work on statistical characterization of filtered phase noise can be found in [26] where most of the work is based on small phase noise assumptions. Work on finding moments of decision variables involving filtered phase noise can be found in [27] and [28]. The PDF of a filtered phase noise random variable is determined in [29] by inverting a small phase noise expression of the MGF. Alternatively, a recursive formula for the moments of filtered phase noise is used in [30] to estimate the PDF. In [31] and [32] an estimated MGF is determined from a simulated population of filtered phase noise samples, leading to power series expansions of the natural logarithm of the MGF.

2 System modeling

Let's consider a binary optical heterodyne FSK communication system employing symbol slicing and majority vote combining. In this system the bit interval T will be split into L chip intervals each equal to $T_{ch} = T/L$. Let's start by defining the two transmitted signals representing binary "1" and "0":

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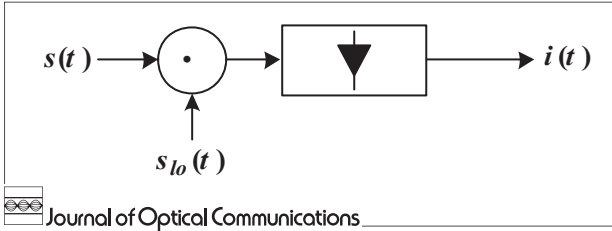


Fig. 1: Heterodyne optical detector

$$s(t) = \begin{cases} s_1(t) = A_s \cos[2\pi f_1 t + \theta_1(t)], & \text{"1"} \\ s_0(t) = A_s \cos[2\pi f_0 t + \theta_0(t)], & \text{"0"} \end{cases} \quad (1)$$

where A_s is the signal amplitude, assumed to be constant, $f_1 = f_c + \Delta f$ and $f_0 = f_c - \Delta f$ are the signal frequencies (centered about f_c), and $\theta_1(t)$ and $\theta_0(t)$ are Wiener-Levy laser phase noise processes, each having a linewidth equal to β_s . The frequency deviation Δf is chosen to make $s_1(t)$ and $s_0(t)$ orthogonal over the chip interval T_{ch} .

On the receiving side, the signal is first detected using a heterodyne optical detector like the one in Fig. 1. The local laser signal required for heterodyne detection is assumed to have the form

$$s_{lo}(t) = A_{lo} \cos[2\pi f_{lo} t + \theta_{lo}(t)] \quad (2)$$

where A_{lo} and f_{lo} are the local laser signal amplitude and frequency, while $\theta_{lo}(t)$ is its phase noise process, assumed to be Wiener-Levy with linewidth β_{lo} . All phase noise processes $\theta_1(t)$, $\theta_0(t)$ and $\theta_{lo}(t)$ will be assumed to be representable in the form [26]

$$\theta(t) = 2\pi \int_0^t \zeta(\tau) d\tau \quad (3)$$

where $\zeta(t)$ is a zero-mean white gaussian random process with double-sided power spectral density $\beta_s/2\pi$ for $\theta_0(t)$ and $\theta_1(t)$, and $\beta_{lo}/2\pi$ for $\theta_{lo}(t)$.

A dual filter configuration of the noncoherent FSK demodulator will be employed. Therefore, the probability

of error analysis is easily seen to be independent of the transmitted symbol. Therefore, all forthcoming analysis will be based on the assumption that a "1" has been transmitted. Without loss of generality, we can ignore the proportionality constants involved in the PD optical-to-electrical conversion process. Hence, it will be assumed that the photodetector output current is given by:

$$i(t) = LP\left\{[s_1(t) + s_{lo}(t)]^2\right\} + x(t) = I_{dc} + A_{lo} A_s \cos[2\pi f_h t + \theta_h(t)] + x(t) \quad (4)$$

where

$$I_{dc} = \frac{A_s^2 + A_{lo}^2}{2} \quad (5)$$

and f_h is the heterodyne frequency given by:

$$f_h = f_{lo} - f_1. \quad (6)$$

$\theta_h(t) = \theta_{lo}(t) - \theta_1(t)$ will be termed the heterodyne phase noise. Owing to the statistical independence of $\theta_1(t)$ and $\theta_{lo}(t)$, it can be easily shown that $\theta_h(t)$ satisfies (3) with a linewidth equal to $\beta = \beta_{lo} + \beta_s$. The PD shot noise current $x(t)$ is additive white gaussian noise (AWGN) with power spectral density equal to $A_{lo}^2/2$ [26]. Defining the intermediate frequency

$$f_{IF} = f_{lo} - f_c \quad (7)$$

and substituting (7) into (6) shows that $f_h = f_{IF} - \Delta f$. It can be easily found that when a "0" is transmitted $f_h = f_{IF} + \Delta f$.

To simplify the remaining analysis, use will now be made of complex (lowpass) representation of narrowband signals taking f_{IF} as the central frequency. Hence, from (4), (6) and (7),

$$\tilde{i}(t) = A_{lo} A_s e^{j[-2\pi\Delta f t + \theta_h(t)]} + \tilde{x}(t) \quad (8)$$

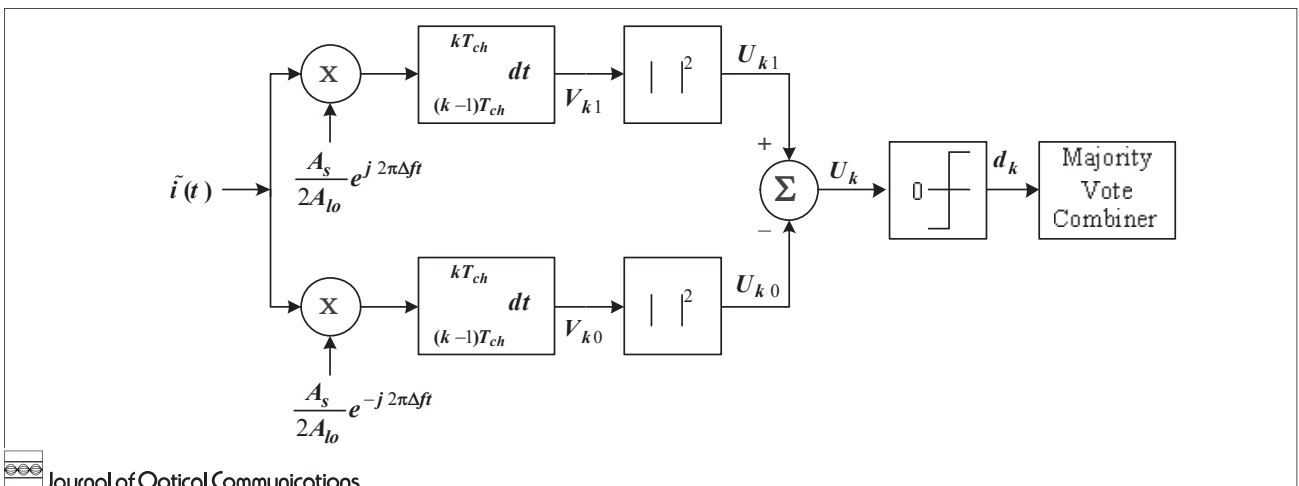


Fig. 2: symbol slicing noncoherent FSK demodulator

where $\tilde{i}(t)$ and $\tilde{x}(t)$ are the lowpass equivalents of $i(t)$ and $x(t)$, respectively. Note that $\tilde{x}(t)$ is AWGN with power spectral density equal to $A_{lo}^2/2$ [26]. Note also that the DC term I_{dc} appearing in has been dropped because it is irrelevant in subsequent analysis.

The proposed demodulator is schematically shown in Fig. 2. It is based on the familiar noncoherent FSK demodulator architecture. The proposed modification is that the bit duration is split into L equal chip durations, each equal to $T_{ch} = T/L$, and a decision is made on each chip. After all chip decisions have been made, a bit decision is made using a majority vote combiner. The slicing correlator outputs can be found as follows:

$$V_{kl} = \frac{A_s}{2A_{lo}} \int_{(k-1)T_{ch}}^{kT_{ch}} \tilde{i}(t) e^{j2\pi(2l-1)\Delta f t} dt, \quad l = 0, 1 \quad (9)$$

where $l = 1$ is used with the upper branch (the “1” branch), while $l = 0$ is used with the lower branch (the “0” branch). Substitution for $\tilde{i}(t)$ yields

$$V_{kl} = \begin{cases} m_{ch} \eta_k + X_{kl}, & l = 1 \\ X_{k0}, & l = 0 \end{cases} \quad (10)$$

where m_{ch} is the chip energy and η_k is a filtered phase noise random variable. m_{ch} and η_k are given by:

$$m_{ch} = \frac{A_s^2 T_{ch}}{2} \quad (11)$$

$$\eta_k = \frac{1}{T_c} \int_{(k-1)T_{ch}}^{kT_{ch}} e^{j\theta_n(t)} dt. \quad (12)$$

The noise output of the PD is given by:

$$X_{kl} = \frac{A_s}{2A_{lo}} \int_{(k-1)T_{ch}}^{kT_{ch}} \tilde{x}(t) e^{j2\pi(2l-1)\Delta f t} dt. \quad (13)$$

It can be easily shown that X_{kl} has a variance $\sigma^2 = m_{ch}/2$ for all values of k and l .

Using the total probability theorem and because of symmetry, we can write the k th chip error probability in the form

$$P(e_k) = \frac{1}{2} [P(e_k/1) + P(e_k/0)] = P(e_k/1). \quad (14)$$

The latter equality in (14) results because the two symbols are assumed equally likely and because $P(e_k/1) = P(e_k/0)$. To determine $P(e_k/1)$, we first note that V_{kl} , V_{k0} , η_k , X_{kl} and X_{k0} in (10) are all complex quantities. In the following analysis, real and imaginary parts of the quantities above will be denoted by appending an R or I, respectively, to the subscript of the involved quantity. Therefore,

$$V_{kl} = (m_{ch} \eta_{kR} + X_{k1R}) + j(m_{ch} \eta_{kI} + X_{k1I}) \\ = V_{k1R} + jV_{k1I} \quad (15)$$

$$V_{k0} = X_{k0R} + jX_{k0I} = V_{k0R} + jV_{k0I}. \quad (16)$$

The squared envelope outputs are thus,

$$U_{k1} = |V_{k1}|^2 = V_{k1R}^2 + V_{k1I}^2 \quad (17)$$

$$U_{k0} = |V_{k0}|^2 = V_{k0R}^2 + V_{k0I}^2. \quad (18)$$

Due to the difficulty in dealing with η_k in random variable transformations, the following derivation of the probability of error will be conditioned on phase noise and will be expressed by making use of (14) written in the form

$$P_\eta(e_k) = P_\eta(e_k/1) \quad (19)$$

where the subscript η denotes statistical conditioning on η . Based on this, each of the random variables V_{k1} and V_{k0} will be seen as being the sum of two independent Gaussian random variables. This leads to the conclusion that U_{k1} and U_{k0} will be chi-squared random variables, each with two degrees of freedom. Let's define

$$q_k^2 = m_{ch}^2 (\eta_{kR}^2 + \eta_{kI}^2) = m_{ch}^2 |\eta_k|^2 \quad (20)$$

Thus, the PDFs of U_{k1} and U_{k0} are, respectively, given by:

$$f_{U_{k1}}(u_1) = \frac{1}{2\sigma^2} e^{-(q_k^2 + u_1)/2\sigma^2} I_0\left(\frac{q_k}{\sigma^2} \sqrt{u_1}\right) \quad (21)$$

$$f_{U_{k0}}(u_0) = \frac{1}{2\sigma^2} e^{-u_0/2\sigma^2} \quad (22)$$

where $I_0(\cdot)$ is the zero order modified Bessel function of the first kind. If “1” is transmitted, a chip error occurs whenever $U_{k0} > U_{k1}$, i. e.,

$$P_\eta(e_k/1) = \Pr\{U_{k0} > U_{k1}/1\}. \quad (23)$$

This is equal to

$$P_\eta(e_k/1) = E_{U_{k1}} [P_\eta(e_k/1, u_1)] \quad (24)$$

where $E_{U_{k1}}[\cdot]$ stands for statistical expectation over U_{k1} . Therefore,

$$P_\eta(e_k/1) = E_{U_{k1}} [1 - F_{U_{k0}}(u_1)] \quad (25)$$

in which $F_{U_{k0}}(\cdot)$ is the cumulative density function (CDF) of U_{k0} . Noting that

$$F_{U_{k0}}(x) = 1 - e^{-x/2\sigma^2} \quad (26)$$

it follows that

$$P_\eta(e_k) = \Psi_{U_{k1}}\left(-\frac{1}{2\sigma^2}\right) \quad (27)$$

where $\Psi_{U_{k1}}(\cdot)$ is the MGF of U_{k1} given by:

$$\Psi_{U_{k1}}(s) = \frac{1}{1 - 2\sigma^2 s} e^{sq_k^2/(1-2\sigma^2 s)} \quad (28)$$

Substituting (28) into (27) results in

$$P_{\eta}(e_k) = \frac{1}{2} e^{-q_k^2/4\sigma^2} = \frac{1}{2} e^{-m_{ch}\Lambda_k/2} \quad (29)$$

where $\Lambda_k = |\eta_k|^2$. To account for averaging over phase noise and determine the unconditional probability of error we write

$$P(e_k) = E_{\Lambda_k} [P_{\eta}(e_k)]. \quad (30)$$

Noting that $\{\Lambda_k, k = 1, 2, \dots, L\}$ are statistically independent identically distributed random variables [26], and applying (30) to (29) yields the chip probability of error

$$P(e_k) = \frac{1}{2} \Psi_{\Lambda} \left(-\frac{m_{ch}}{2} \right) \quad (31)$$

where $\Psi_{\Lambda}(\cdot)$ is the MGF of Λ_k (or simply Λ_1 , because the MGF is equal for all $\{\Lambda_k\}$). Note that (31) shows that all chip error probabilities are equal. To emphasize that, we'll denote a chip error probability by

$$P(e_k) = P(e_{ch}), \quad k = 1, 2, \dots, L. \quad (32)$$

It was shown in [31] and [32] that the moment generating function in (31) can be put in the form

$$\Psi_{\Lambda}(s) = \exp \left(\sum_{n=0}^N a_n [g(s)]^n \right) \quad (33)$$

where $\{a_n, n = 1, 2, \dots, N\}$ are a set of real-valued coefficients. According to [31] $g(s) = s$, while according to [32],

$$g(s) = \text{sgn}(s) |s|^{1/4}. \quad (34)$$

Since the MGF model in [32] is more accurate than the one in [31], we'll follow the former here, taking into account that $a_0 = 0$ in this model. Substituting (34) and (33) into (31) gives

$$P(e_{ch}) = \frac{1}{2} \exp \left(\sum_{n=1}^N (-1)^n a_n \left(\frac{m_{ch}}{2} \right)^{n/4} \right) \quad (35)$$

Many of the results to be presented later in this paper will be based on the MGF characterizations in [32], which are based on the random variable $\Lambda = |\eta|^2$, where

$$\eta = \frac{1}{T} \int_0^T e^{j\theta(t)} dt \quad (36)$$

where $\theta(t)$ has a linewidth β . However, the MGFs needed in this paper are generally based on a random variable of the form

$$\eta_1 = \frac{1}{T_{ch}} \int_0^{T_{ch}} e^{j\theta(t)} dt \quad (37)$$

Noting that $\theta(t)$ is zero-mean gaussian with a variance of $2\pi\beta t$, and after some simple manipulation it can be shown that η_1 can be computed from (36) by replacing $\theta(t)$ by another phase noise process with a linewidth of

β/L . With this in mind, the coefficients $\{a_n\}$ are generated as in [32] with linewidth-duration product βT replaced by βT_{ch} .

The majority vote device will make a decision in favor of "1" if more than half the chip decisions are in favor of "1", otherwise, it will decide in favor of "0". Hence, a bit error occurs if more than half the chip decisions are in error. Therefore,

$$P(e) = \sum_{i=(L+1)/2}^L \binom{L}{i} (P(e_{ch}))^i (1 - P(e_{ch}))^{L-i} \quad (38)$$

3 BER results

The effect of using symbol slicing and majority vote combining on idealized systems that are free of phase noise is illustrated in Fig. 3. As can be clearly seen, the receiver performance is degraded by the use of symbol slicing and majority vote combining. This is in agreement with the well-known theory of signal detection in AWGN which leads to a purely matched filter structure of the optimum demodulator. Note that the $P(e) = 10^{-9}$ line is also drawn in Fig. 3 to help distinguish regions of acceptable performance. This line is included in all other probability of error graphs. In more realistic situations, laser phase noise is present to a certain extent. The linewidth duration product is a good indication of phase noise severity. Figs. 4–8 illustrate the performance of receivers with linewidth durations between 0.5 and 10. This covers a significant range of the severity of phase noise. The performance improvement due to the use of symbol slicing and majority voting is quite visible in these curves. However, as L is increased beyond the values in these curves, the BER tends to increase. This behavior is illustrated in Fig. 9 which shows optimum values of L for different linewidth duration products. As this figure demonstrates, larger values of optimum L are required when phase noise is more severe. Finally, Figs. 10 and 11 show the receiver performance degradation due to increasing levels of phase noise. Note that the

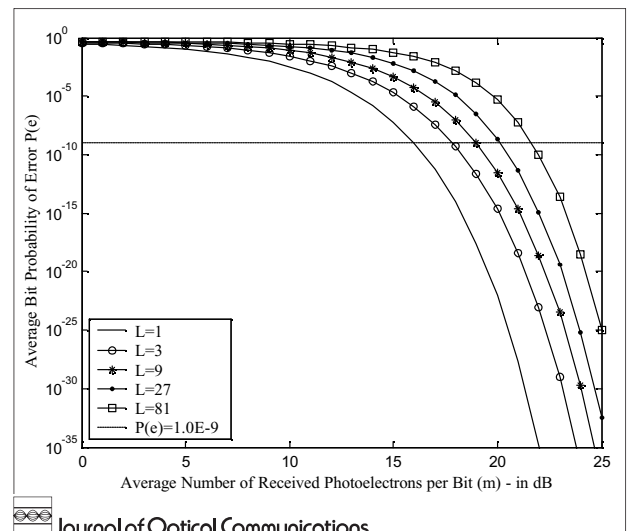


Fig. 3: Performance of a majority vote combining symbol slicing non-coherent FSK optical heterodyne receiver ignoring phase noise

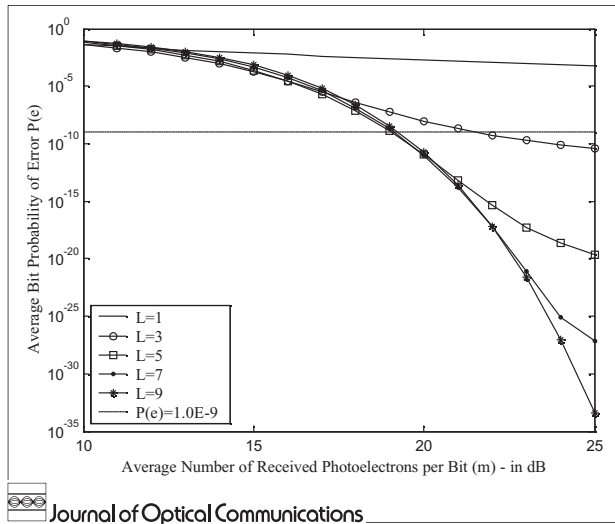


Fig. 4: Performance of a majority vote combining symbol slicing non-coherent FSK optical heterodyne receiver for $\beta T = 0.5$

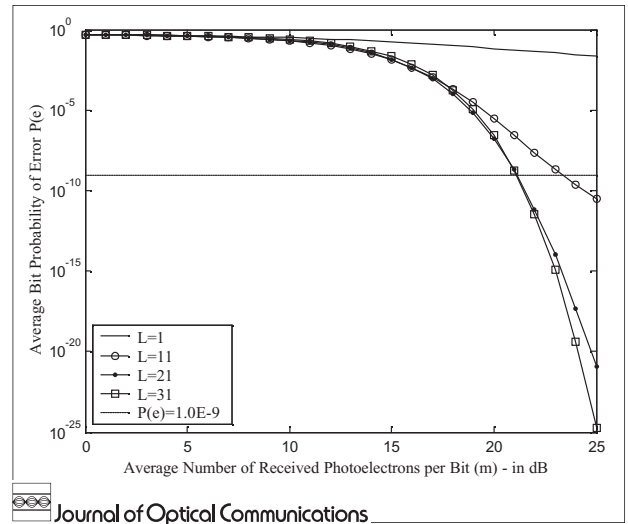


Fig. 7: Performance of a majority vote combining symbol slicing non-coherent FSK optical heterodyne receiver for $\beta T = 5$

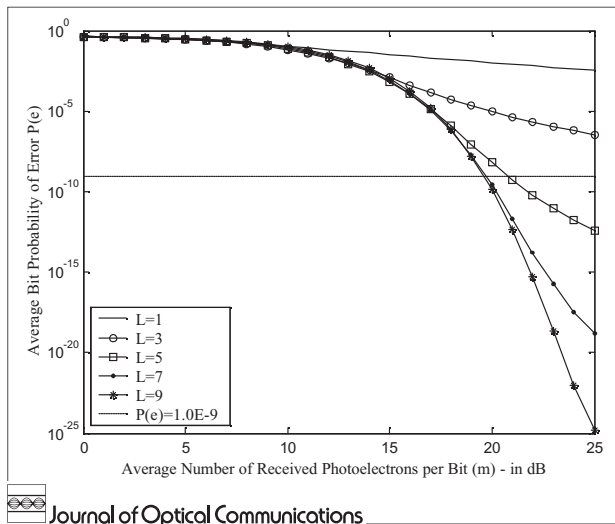


Fig. 5: Performance of a majority vote combining symbol slicing non-coherent FSK optical heterodyne receiver for $\beta T = 1$

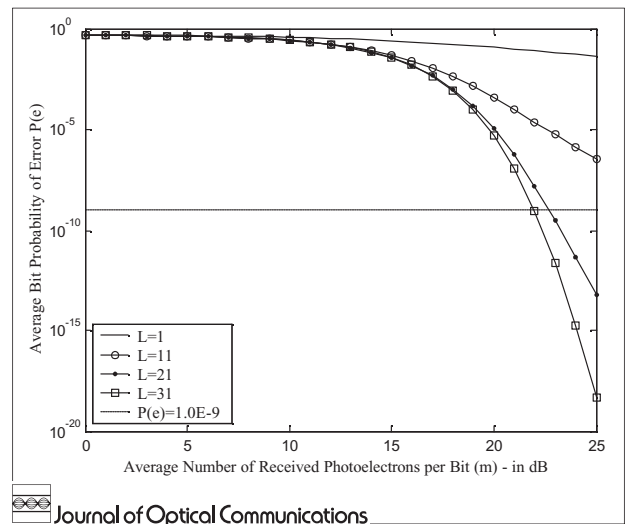


Fig. 8: Performance of a majority vote combining symbol slicing non-coherent FSK optical heterodyne receiver for $\beta T = 10$

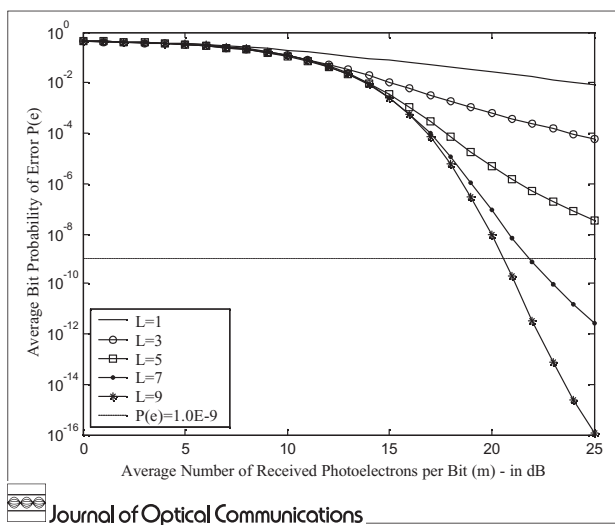


Fig. 6: Performance of a majority vote combining symbol slicing non-coherent FSK optical heterodyne receiver for $\beta T = 2$

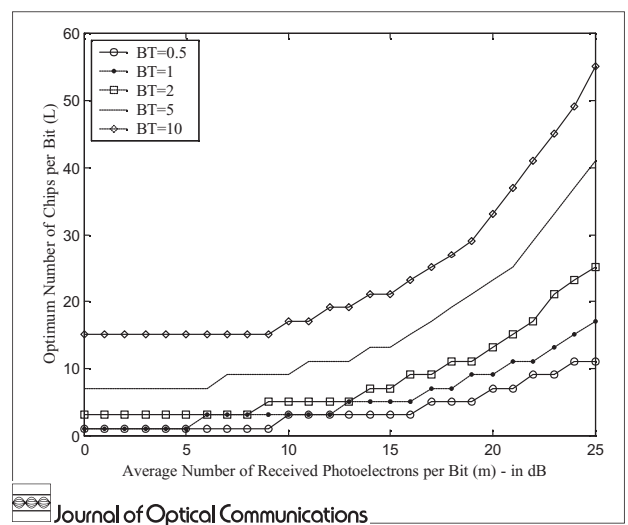


Fig. 9: Optimum values of the number of chips per bit (L) for different linewidth duration products

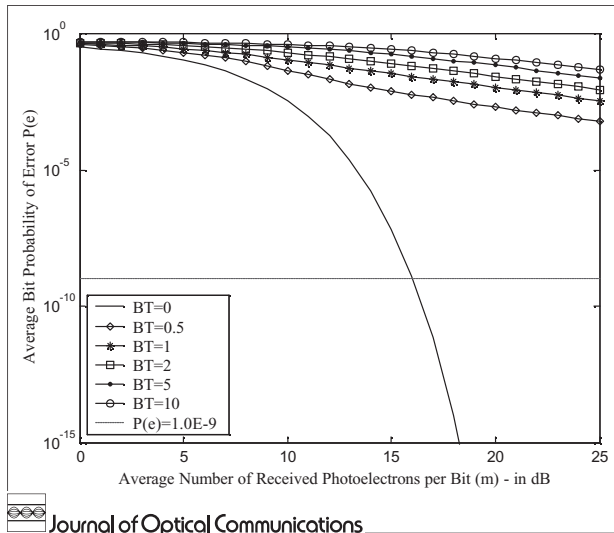


Fig. 10: Performance degradation of a noncoherent FSK optical heterodyne receiver due to laser phase noise (no symbol slicing)

performance degradation is much more significant in the non-symbol slicing receiver of Fig. 10.

The results presented here do not imply that the use of majority voting is the optimum way of combining chip decisions to arrive at bit decisions. Other combining methods may very well give better results. It should be stressed however, that the idea of symbol slicing opens a new domain for receiver optimization in the presence of phase noise. This idea has been previously used in [26]. However, the BER curves in [26] are based on small phase noise approximations even in cases when phase noise is not actually small. That's basically why our results cannot be directly compared to those in [26].

4 Conclusions

This paper has demonstrated two important results. The first is the applicability of phase noise characterization models proposed by the author in [31] and [32] to BER computations in noncoherent optical receivers. These models are accurate enough not to suffer from small phase noise approximation problems usually found in the literature. The second result is the effectiveness of using symbol slicing and majority vote combining in reducing phase-noise-induced performance degradation of such receivers.

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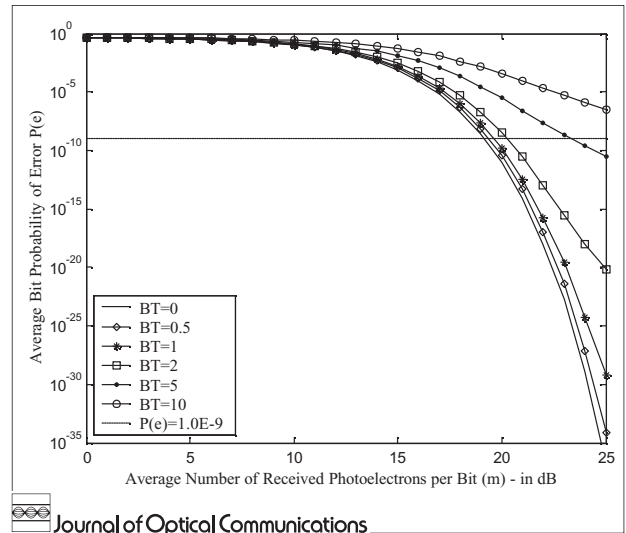


Fig. 11: Performance degradation of a majority vote combining symbol slicing noncoherent FSK optical heterodyne receiver with $L = 11$ due to laser phase noise

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