Performance Analysis of Binary Optical Homodyne ASK with Least Squares Detection

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ABSTRACT

A new method to detect binary optical homodyne amplitude shift keying (ASK) signals in the presence of laser phase noise and receiver shot noise is presented. This method is called the least squares (LS) method. An approximate expression for the error probability is derived using the Gaussian approximation. The bit error rate (BER) performance results are shown for different levels of phase noise.

I. INTRODUCTION

The most widely-used modulation/demodulation scheme in present optical fiber communication systems is the so-called intensity modulation/direct detection (IM/DD) [1]. Intensity modulation (IM) means that the light intensity (not the amplitude) is modulated linearly with respect to the input signal. IM is easy to implement with light emitting diodes (LEDs) or injection lasers. These devices can be directly modulated by variation of their drive currents [2]. Direct detection (DD) means that the optical signal is detected directly at the optical stage of the receiver without frequency conversion (heterodyne/homodyne). Direct detection ignores phase information in the incoming optical signal. Hence, DD is considered to be a noncoherent detection scheme [1].

Coherent (heterodyne or homodyne) optical detection offers better error rate performance compared to direct detection. The performance advantage of coherent detection over DD is mainly due to two factors: the gain resulting from the use of a high-power local laser signal at the receiving side, and the employment of sophisticated modulation techniques [3]. The trend in modern communication systems is towards the use of digital modulation techniques such as phase shift keying (PSK), amplitude shift keying (ASK) and frequency shift keying (FSK) which are possible in coherent detection systems. Variations of these basic techniques can also be used, often resulting in substantial performance advantages.

In spite of these advantages, coherent reception is rather complicated because a strong local laser optical signal is required. The polarization state of the local laser must be perfectly matched to that of the received signal. This condition usually requires a sophisticated polarization control scheme. Alternatively, polarization diversity can be utilized [4].

The performance of coherent optical systems in the presence of both additive shot noise and laser phase noise has been the subject of extensive studies. The current state of this research is that accurate performance evaluations can be made for different modulation schemes using approaches that don't take into account the phase noise process [3].

Foschini and Greenstein [5] considered both shortening the bit period and using a postdetection lowpass filtering for FSK systems. Their results showed that performance gains could be achieved by both approaches. Postdetection filtering has also been considered by Jacobsen and Garrett [6]. On a different track, Dallal and Shamai [7] used another approach to improving differential phase shift keying (DPSK) performance by repeated transmissions of a signal over several chips in one bit period.

A majority logic decoding technique for homodyne PSK receiver was used by Irshid and Kavehrad [8]. In this approach the receiver divides the original bit interval into N subintervals and decides whether the signal in each subinterval is +1 or –1 using an integrate and dump filter followed by a sign detector. By selecting N to be odd, a majority logic decoder is used to make the final decision based on the outcomes of the decisions on the subintervals.

In this paper, a new detection method called least squares (LS) will be investigated. This method will be applied to a binary homodyne ASK system corrupted by both receiver shot noise and lasers phase noise. The BER as a function of the average number of photoelectrons per bit for ASK using LS detection for different levels of phase noise will be shown and compared with the ideal case analysis (assuming no phase noise).

II. COHERENT ASK RECEIVER DESCRIPTION

In ASK the binary zero "0" and one "1" are represented by signals with different amplitudes. Let's consider signals with constant amplitudes and time durations T in the form [9]:

$$s(t) = \begin{cases} s_1(t) = A\cos\left[\omega_0 t + \theta_1(t)\right], & 1 \\ s_0(t) = 0, & 0 \end{cases}, \quad 0 \le t < T \quad (1)$$

where A is the signal amplitude assumed to be constant, and ω_0 is the optical angular frequency. In coherent detection, the received signal is added optically to a local laser (LO) signal with constant amplitude and with frequency and polarization identical to those of the transmitted signal:

$$s_{LO}(t) = C\cos\left[\omega_0 t + \theta_0(t)\right] \quad (2)$$

The combined signal is focused on a photodetector (PD), which produces a shot noise current random process. Assuming bit "1" is transmitted, the output shot noise current after normalizing the PD responsivity R_p [3], can be written as [4]

$$i_1(t) = \mathbf{LP}\left\{\left|S_1(t) + S_{LO}(t)\right|^2\right\} + X_1''(t) \quad (3)$$

where $\mathbf{LP}\{\ \}$ means taking the lowpass part and $X_1''(t)$ is the PD shot noise modeled as a WGN process with zero mean and variance equal to $C^2/2$ [9]

After simple manipulation equation (3) can be written as

$$i_1(t) = \frac{C^2 + A^2}{2} + CA\cos\left[\theta(t)\right] + X_1''(t) \quad (4)$$

where $\theta(t) = \theta_1(t) - \theta_0(t)$ is the difference between the phase noise of the transmitting laser and the local laser which is a Wiener-Levy process. $\theta(t)$ can be approximated by non-stationary Gaussian process [9] with variance equal to $2\pi\beta_L t$, where β_L is the 3 dB total lasers linewidth. By using the same approach the PD output current assuming bit "0" is transmitted can be written as

$$i_0(t) = \frac{C^2}{2} + X_0''(t) \quad (5)$$

where $X_0''(t)$ is WGN (independent of $X_1''(t)$) with variance $C^2/2$ [9].

III. HOMODYNE ASK RECEIVER WITH LS DETECTION

The LS detection scheme is suggested to reduce the effects of phase noise in coherent optical receivers. The idea of this scheme is very simple and is based on dividing the photodetector output signal into two separate paths and subtracting the expected ideal signal for the data bit "1" from one path and the expected ideal signal for the data bit "0" from the other path. The term ideal signal refers to the signal without laser phase noise or PD shot noise processes.

The error signals from each path are passed through bandpass filters centered on the heterodyne intermediate frequency (matched filters). The filter outputs are sampled L times during the bit duration time T. Each sample is squared and the samples corresponding to one bit are added together in each path.

The path that gives the lower output contains the correct ideal signal and so a decision can be easily taken. It should be easy to figure out that LS detection as proposed in this paper is a form of time diversity. Furthermore, this method can be used in any digital communication system.

According to the description above and using equations (4) and (5) the block diagram for a coherent homodyne ASK receiver with LS detection is shown in Fig. 1.

Since C is a common factor in the PD current the PD current is divided by this factor before processing. Hence, the new approximate current variables assuming that $C >> A$ can be written as

$$I_i(t) \approx \frac{C}{2} + A_i\cos\left[\theta(t)\right] + X_i'(t) \qquad i = 1, 0 \quad (6)$$

where $A_1 = A$, $A_0 = 0$, and $X_i'(t) = X_i''/C$ are independent WGN processes with variance equal $1/2$.

The lowpass filter $\mathbf{H}$ is simply an integrate and dump filter centered at the homodyne frequency. Its impulse response is

$$h(t) = \begin{cases} \dfrac{1}{T'} & ; \quad 0 \le t \le T' \\ 0 & ; \quad otherwise \end{cases} \quad (7)$$

During the data bit interval the lowpass filter output is sampled L times at $t = kT'$, $k = 1, 2, \ldots\ldots, L$, generating a sequence of random variables. Assuming "1" is transmitted, then

$$V_{uk} = \frac{1}{T'}\int_{(k-1)T'}^{kT'} \left(I_1(t) - A - C/2\right)\, dt = \frac{1}{T'}\int_{(k-1)T'}^{kT'} \left(A\cos[\theta(t)] + X'_1(t) - A\right)\, dt \qquad (8)$$

and

$$V_{lk} = \frac{1}{T'}\int_{(k-1)T'}^{kT'} (I_1(t) - C/2)\, dt = \frac{1}{T'}\int_{(k-1)T'}^{kT'} \left(A\cos[\theta(t)] + X_1'(t)\right)\, dt \qquad (9)$$

The decision variable U_1 assuming "1" transmitted and according to Fig.1 is

$$U_1 = \frac{T'}{2}\sum_{k=1}^{L}\left[\left|V_{uk}\right|^2 - \left|V_{lk}\right|^2\right] \qquad (10)$$

where the factor $T'/2$ is included to simplify the results. The fact that $X(t)$ is a white Gaussian noise and the phase noise $\theta(t)$ a random walk process with independent increments makes $\left|V_{uk}\right|^2 - \left|V_{lk}\right|^2$ a sequence of independent and identical random variables [9],so U_0 is a set of L independent and equally distributed random variables , dropping the index k and after simple calculations

$$U_1 = \frac{T'}{2}\sum^{L}\left(A^2 - 2A^2 y_c - 2AX_1\right) \qquad (11)$$

where

$$X_1 = \frac{1}{T'}\int_0^{T} X'_1(t)\, dt \qquad (12)$$

is the filtered shot noise process with zero mean and variance equal $1/2T'$.

and

$$y_c = \frac{1}{T'}\int_0^{T'} \cos[\theta(t)]\, dt \qquad (13)$$

is the filtered phase noise process with mean and variance that can be calculated as follows:

$$E[y_c] = \frac{2}{B'}\left(1 - e^{-B'/2}\right) \qquad (14)$$

where $B' = 2\pi\beta_L T'$, β_L is the total transmitter and LO 3-dB linewidth, and

$$Var[y_c] = \frac{1}{3B'^2}\Re(B') \qquad (15)$$

where

$$\Re(B') = \left(e^{-2B'} - 12e^{-B'} + 32\, e^{-B'/2} + 6B' - 21\right) \qquad (16)$$

When bit "0" is transmitted no signal component will appear in the PD current, thus,

$$V_{uk} = \frac{1}{T'}\int_{(k-1)T'}^{kT'} \left(I_0(t) - A - C/2\right)\, dt = \frac{1}{T'}\int_{(k-1)T'}^{kT'} \left(X'_0(t) - A\right)\, dt \qquad (17)$$

and

$$V_{lk} = \frac{1}{T'}\int_{(k-1)T'}^{kT'} (I_0(t) - C/2)\, dt = \frac{1}{T'}\int_{(k-1)T'}^{kT'} \left(X'_0(t)\right)\, dt \qquad (18)$$

The new decision variable U_0 is simply equal to U_1 without the signal component $-2Ay_c$, therefore,

$$U_0 = \frac{T'}{2}\sum^{L}\left(A^2 - 2AX_0\right) \qquad (19)$$

To simplifiy the calclations we assume that the decision variables U_1 and U_0 have Gaussian distributions with a decision threshold α. Assuming that η_1, η_0 are the means, and σ_1, σ_0 are the standard deviations of U_1 and U_0, respectively, the BER can be expressed in the form [9]:

$$P_e = \frac{1}{2}Q\left(\frac{\eta_0 - \alpha}{\sigma_0}\right) + \frac{1}{2}Q\left(\frac{\alpha - \eta_1}{\sigma_1}\right) \qquad (20)$$

where Q refers to the Q-function

The decision variables for homodyne ASK are given in equations (11) and (18), their means and variances can be determined as follows:

$$\eta_1 = E[U_1] = \frac{T'}{2}\sum^{L} E\left[A^2 - 2A^2 y_c - 2AX_1\right] = \frac{T}{2}\left\{A^2 - \frac{2A^2.2}{B'}\left(1 - e^{-B'/2}\right)\right\} \qquad (21)$$

Letting, $m_1 = A^2 T/2$ which represents the expected number of photoelectrons assuming "1" is transmitted, equation (20) can be rewritten as

$$\eta_1 = m_1 - \frac{4m_1}{B'}\left(1 - e^{-B'/2}\right) \qquad (22)$$

$$\sigma_1^2 = Var[U_1] = \frac{T'^2}{4}\sum^{L} Var\left[A^2 - 2A^2 y_c - 2AX_1\right] = \frac{T^2}{4L}\left\{\frac{4A^4}{3B'^2}\Re(B') + \frac{4A^2}{2T'}\right\} \qquad (23)$$

which can be written as

$$\sigma_1^2 = \frac{4m_1^2}{3LB'^2}\Re(B') + m_1 \qquad (24)$$

Now, assuming "0" is transmitted,

$$\eta_0 = E[U_0] = \frac{T'}{2}\sum^{L} E\left[A^2 - 2AX_0\right] = \frac{A^2T}{2} = m_1 \tag{25}$$

and

$$\sigma_0^2 = Var[U_0] = \frac{T'^2}{4}\sum^{L} Var\left[A^2 - 2AX_0\right] = \frac{T^2}{4L}\cdot\frac{4A^2}{2T'} = m_1 \tag{26}$$

By substituting equations (21), (23), (24) and (25) into equation (19) the BER for homodyne ASK optical system can be written as

$$P_e = \frac{1}{2}Q\left(\sqrt{m_0}\right) + \frac{1}{2}Q\left(\frac{-m_0 + \frac{4m_0}{B'}\left(1 - e^{-B'/2}\right)}{\sqrt{\frac{4m_0^2}{3LB'^2}\Re(B') + m_0}}\right) \tag{27}$$

IV. RESULTS AND DISCUTION

A simple MATLAB program is used to plot P_e versus the avearage number of photoelectrons per bit ($m_1/2$) for a phase noise parameter $\beta_L T = 0.1, 1, 10$ using different values of L.

It's easy to note that as the number of segmentations of the bit duration L is increased we get a better performance (the required number of photoelectrons to get a certain P_e is decreased). This improvement in performance is due to the fact that the random phase process changes slowly during a small integration period.

For $\beta_L T = 1$ it is expected that; since the noise variance will increase; we must increase the number of segmentations L in order to get an acceptable performance, the result is shown in Fig.3

Receiver performance using LS detection is very poor using $L = 1$. This fact shows how the laser phase noise has a major effect on the performance of coherent optical transmission systems [8].

As can be seen from the Figures, the ideal case performance (assuming no phase noise) using a matched filter as in [9] can be obtained even if the phase noise parameter is large by increasing the bit segmentations L using the proposed LS method but up to certain limits. This is because; as shown in the figures; for large values of L the phase noise process becomes constant and the probability of error systems performance changes slowly as L is increased.

V. CONCLUTIONS

The results show that the LS detection method is a suitable detection scheme for coherent optical ASK homodyne systems courrepted by laser phase noise and PD shot noise. With LS detection the ideal receiver sensitivity can be obtained. Anthor advantage is that the threshold of the decision vvariables is equal to zero using the LS method and no effort must be done to select an optimal threshold that minimizes thr probability of error.

VI. REFERENCES

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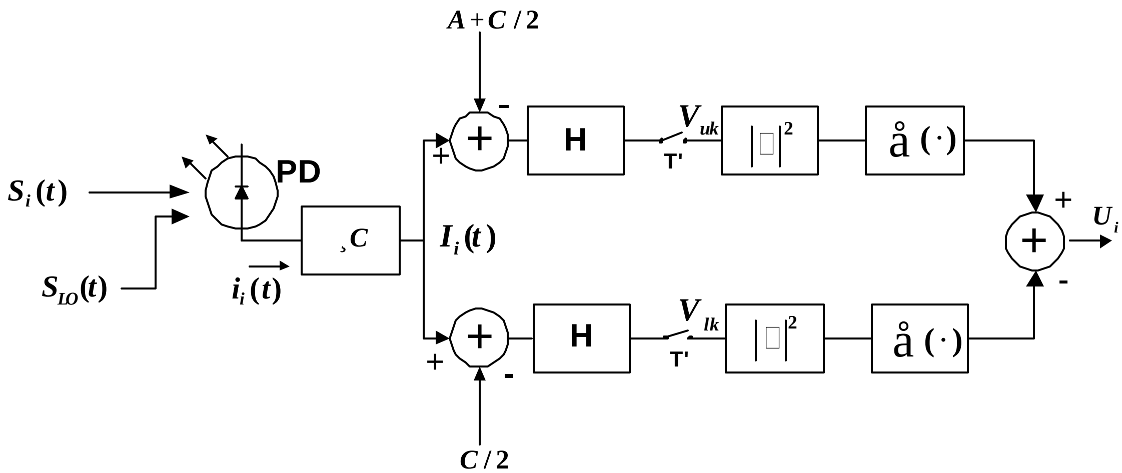

Fig.1: Homodyne ASK receiver with LS detection. ***H*** is a lowpass filter

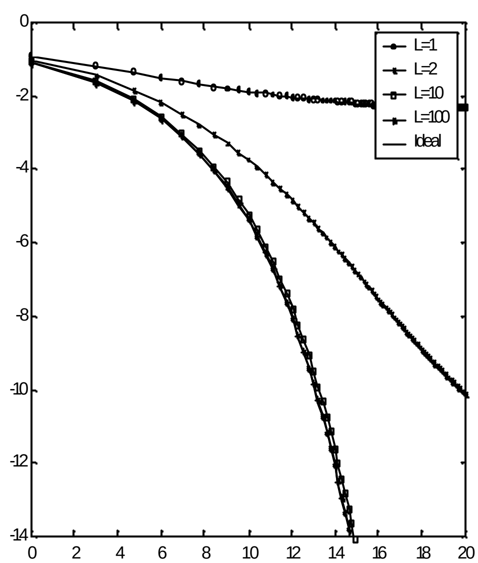

Fig.2: BER for homodyne LS ASK calculated by the Gaussian approximation versus the average number of photoelectrons per bit $m = (m_0 + m_1)/2 = m_1/2$. The phase noise parameter $\beta_L T = 0.1$ and $L = 1, 2, 10, 100$

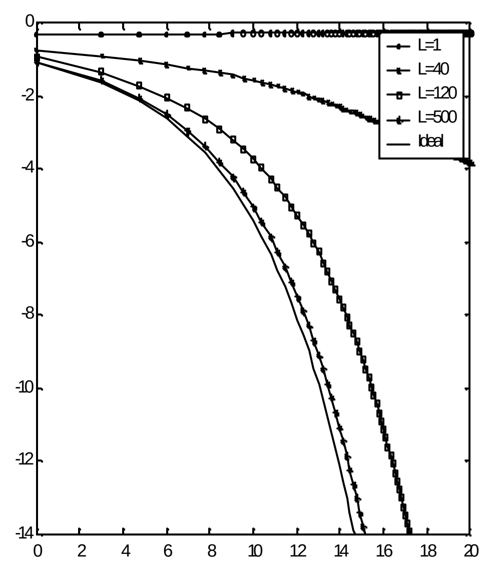

Fig.3: BER for homodyne LS ASK calculated by the Gaussian approximation versus the average number of photoelectrons per bit $m = m_1/2$. The phase noise parameter $\beta_L T = 1$ and $L = 1,10,40,250$.

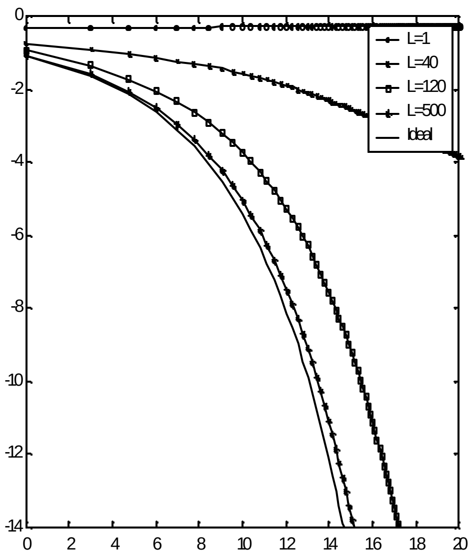

Fig.4: BER for homodyne LS ASK calculated by the Gaussian approximation versus the average number of photoelectrons per bit $m = m_1 / 2$. The phase noise parameter $b_L T = 10$ and $L = 1, 40, 120, 500$.