Performance Evaluation of Heterodyne Optical Multi-Carrier MFSK Based on BIB Design

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Abstract- **Performance of a noncoherent heterodyne optical multi-carrier frequency shift keying (MC-MFSK) system is evaluated in the presence of laser phase noise and photodetector shot noise. Balanced incomplete block (BIB) design from combinatorial theory is utilized to reduce the number of lasers and wavelengths needed to construct the MFSK symbols. To attain the best possible performance optical heterodyne detection is employed in the receiver. Receiver performance is evaluated in terms of a union upper bound on the bit error rate (BER). A closed form expression for the union bound on the error probability conditioned on laser phase noise is obtained. The result was averaged over laser phase noise using simulation. It can be seen from the BER results that when laser phase noise is significant MC-MFSK is several dB better than conventional MFSK, owing to the implicit frequency diversity that is present in the former. This demonstrates that the effects of laser phase noise are substantially reduced due to the implicit diversity.**

I. INTRODUCTION

It is well-known that semiconductor laser phase noise is a major source of performance degradation of optical receivers [1-4]. It can be anticipated that using several replicas of a signal that have been subjected to independent phase noise processes can improve the receiver ability to restore the information, and hence, improve the system bit error rate performance.

Using BIB design [5], [6] a number of distinct orthogonal tones $v < M$ is used to represent the M MFSK symbols. Each MFSK symbol is transmitted using $w << v$ tones. The w tones could be transmitted sequentially or simultaneously. In the system proposed in this paper tones are transmitted simultaneously with symbol energy divided equally among them. The v tones are used to produce $b \geq M$ signals such that each signal contains exactly w distinct tones. Each one of the v tones occurs in exactly r signals. Every distinct pair of tones occurs in exactly one signal.

From combinatorial theory [5], [6] the admissible values of v for w=3, 4 required to represent M symbols in MC-MFSK are tabulated in Table 1. Note that the table includes other quantities that will be defined later in this paper.

As a result of the above, using BIB design to construct the MFSK signals creates implicit frequency diversity which is known to improve power efficiency without the customary bandwidth expansion introduced by the use of diversity [5].

TABLE I
ADMISSIBLE VALUES OF v FOR w=3,4

	MC(v,k,3)				MC(v,k,4)				MFSK
k	v	b	l_N	l_I	v	b	l_N	l_I	M
1	7	7	0	6	13	13	0	12	2
2	7	7	0	6	13	13	0	12	4
3	9	12	2	9	13	13	0	12	8
4	13	26	10	15	16	20	3	16	16
5	15	35	16	18	25	50	21	28	32
6	21	70	42	27	37	111	66	44	64
7	31	155	112	42	40	130	81	48	128
8	43	301	240	60	61	305	228	76	256

Besides, in MC-MFSK the number of tones used to represent the symbols is reduced compared to conventional MFSK ($v < M$). Hence, the proposed multi-carrier system is supposed to improve both power and bandwidth efficiencies compared to conventional MFSK systems.

This paper is organized as follows: Section II contains signal analysis and modeling. Section III introduces the receiver structure and BER analysis. Numerical results and comparisons are presented in section IV. Finaly conclusions are drawn in section V.

II. SIGNAL ANALYSIS AND MODELING

In the proposed system w tones are used to represent each symbol of duration T. To represent the tone assignment to symbols a group $G_i, i = 1, 2, \ldots, M$ containing w tone numbers is associated with each symbol. Hence the transmitted signal representing the ith symbol has the form:

$$s_i(t) = \sum_{\substack{k=1 \\ k \in G_i}}^{v} x_k(t), \quad i = 1, 2, \ldots, M, \quad 0 \leq t < T, \tag{1}$$

where $x_k(t)$ represents the waveform for tone number k which is given by:

$$x_k(t) = A\cos[2\pi F_k t + \theta_k(t)], \tag{2}$$

in which F_k is the tone frequency, $\theta_k(t)$ is the laser phase noise process modeled as a Wiener-Levy process with linewidth α_s. The average number of photons representing the energy of this tone at the photodetector (PD) input is proportional to:

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$$m_t = \frac{A^2 T}{2}. \tag{3}$$

Accordingly, the average number of received photons reprtesenting a symbol is proportional to $m_s = w m_t$, while the average number of received photons representing a bit is proportional to:

$$m_b = \frac{m_s}{K} = \frac{w}{K} m_t, \tag{4}$$

where K is the number bits per symbol. Tone frequencies are chosen to satisfy:

$$F_k = f_0 + f_k, \tag{5}$$

where f_0 is the optical center frequency, and f_k is a frequency increment (or decremnt) representing the used tone. Adjacent tone frequencies are separated by a spacing $2f_d$ chosen to make them orthogonal. Hence, f_k can takes its value from the set $\{(2k-1-v)f_d$, where $k = 1, 2, \ldots, v$.

The front-end of the receiver is a heterodyne optical detector, as illustrated in Fig. 1. Assuming symbol i is transmitted, the signal present at the PD input is given by:

$$r_i(t) = s_i(t) + s_{lo}(t), \tag{6}$$

where the local laser signal $s_{lo}(t)$ is given by:

$$s_{lo}(t) = C\cos(2\pi f_0 t + \theta_{lo}(t)), \tag{7}$$

where $\theta_{lo}(t)$ is the local laser phase noise process having linewidth α_{lo}. Given that $C \gg A$, assuming unity PD responsivity and ignoring DC terms, the generated PD current is given by:

$$\begin{aligned} I_i(t) &= \mathrm{LP}\{r_i^2(t)\} + I_n(t) \\ &= AC \sum_{\substack{k=1 \\ k \in G_i}}^{w} \cos[2\pi f_k t + \varphi_k(t)] + I_n(t) \end{aligned} \tag{8}$$

where the LP operator sands for taking the lowpass part, $\varphi_k(t) = \theta_k(t) - \theta_{lo}(t)$ is a phase noise process with linewidth $\alpha_s + \alpha_{lo}$, and $I_n(t)$ is the PD shot noise current process assumed to be white gaussian with two-sided power spectral density equal to $C^2/2$.

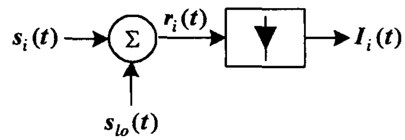

Fig. 1: Heterodyne optical detection

III. RECEIVER STRUCTURE AND BER ANALYSIS

The PD current is used in the noncoherent FSK demodulator illustrated in Fig. 2. The noncoherent demodulator starts with a bank of v matched filter square-law detectors. The function of these detectors is given by:

$$\beta_l = \left| \frac{A}{C}\int_0^T I_i(t)\cos(2\pi f_l t)dt \right|^2 + \left| \frac{A}{C}\int_0^T I_i(t)\sin(2\pi f_l t)dt \right|^2 \tag{9}$$

where f_l is any one of the v tones. It can be easily found that:

$$\beta_l = \begin{cases} (m_t\gamma_{lc} + N_{lc})^2 + (m_t\gamma_{ls} + N_{ls})^2, & l \in G_i \\ N_{lc}^2 + N_{ls}^2, & l \notin G_i \end{cases}, \tag{10}$$

where

$$\gamma_{lc} = \frac{1}{T}\int_0^T \cos[\varphi_l(t)]dt, \tag{11}$$

$$\gamma_{ls} = \frac{1}{T}\int_0^T \sin[\varphi_l(t)]dt, \tag{12}$$

and

$$N_{lc} = \frac{A}{C}\int_0^T I_n(t)\sin(2\pi f_l t)dt, \tag{13}$$

$$N_{ls} = \frac{A}{C}\int_0^T I_n(t)\sin(2\pi f_l t)dt. \tag{14}$$

Note that N_{lc} and N_{ls} can both be shown to be zero-mean white gaussian processes with power spectral densities equal to:

$$\sigma_n^2 = \frac{m_t}{2} \tag{15}$$

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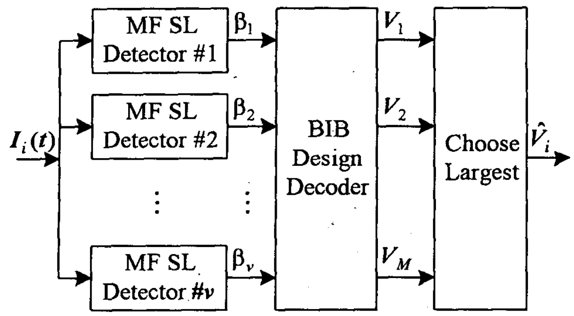

Fig. 2: Noncoherent FSK demodulator

The tones comprising $s_i(t)$ have frequencies $f_0 + f_k$, $k \in G_i$. From Fig. 2. the probability of a correct decision is found from:

$$P_c = \Pr\left(V_i = \max_{m=1,2,\dots,M} \left[V_m \middle| s_i(t) \right) \right). \quad (16)$$

Note that

$$V_m = \sum_{\substack{j=1 \\ j \in G_m}}^{v} \beta_j , \quad (17)$$

where the β_j 's are taken from the outputs of the w square-law detectors matched to the tones of $s_m(t)$. Now, the decision variables $V_m, m \neq i$ can be categorized into two sets relative to V_i. The first set is called the nonintersecting set with l_N elements $V_1', V_2', \dots, V_{l_N}'$. Within this set there is no decision sample β_j in common to V_i. The second set is called the intersecting set with l_I elements $V_1'', V_2'', \dots, V_{l_I}''$. This set has only one decision sample β_j in common to V_i.

A union bound on the symbol probability of error conditioned on laser phase noise can be shown to be given by:

$$P_e \leq \sum_{k=1}^{l_N} \Pr\left(V_i < V_k' \middle| s_i \right) + \sum_{k=1}^{l_I} \Pr\left(V_i < V_k'' \middle| s_i \right). \quad (18)$$

Note that $\Pr\left(V_i < V_k' | s_i \right), k = 1, 2, \dots, l_N$ are equal. Similarly, $\Pr\left(V_i < V_k'' | s_i \right), k = 1, 2, \dots, l_I$ are equal. Hence,

$$P_e \leq l_N \Pr\left(V_i < V_k' \middle| s_i \right) + l_I \Pr\left(V_i < V_k'' \middle| s_i \right). \quad (19)$$

After lengthy derivation it was found that

$$\Pr\left(V_i < V_k' \middle| s_i \right) = \frac{1}{2^{2w-1}} e^{-\gamma/2} \sum_{k=0}^{w-1} d_k \left(\frac{\gamma}{2} \right)^k , \quad (20)$$

where

$$d_k = \frac{1}{k!} \sum_{n=0}^{w-1-k} \binom{2w-1}{n}, \quad (21)$$

$$\gamma = m_b \frac{K}{w} \sum_{\substack{k=1 \\ k \in G_i}}^{v} \left[\gamma_{kc}^2 + \gamma_{ks}^2 \right]. \quad (22)$$

The probability in the second part of (19) (i.e., the intersecting set) can be found to have the same form of eq. (20) with w replaced by w-1.

The union bound on the average symbol probability of error is found by averagig (19) over γ yielding:

$$P_E = E_\gamma \left[P_e \right]. \quad (23)$$

Finally, the average bit probability of error P_b, is related to P_E according to:

$$P_b = \frac{2^{K-1}}{2^K - 1} P_E . \quad (24)$$

The averaging operation in (23) was performed using simulation on Matlab.

IV. NUMERICAL RESULTS

As a shorthand notation, MC-MFSK system based on BIB design is represented as MC(v, K, w). The conventional MFSK is represented by FSK(M, K). The bit probability of error versus the average number of photons per bit has been calculated numerically for the MC$(v,8,w)$ system with w=3,4 and duration-linewidth product (BT) equal to 0, 10 and 25. These data are compared with the FSK(256,8) system. The results have been plotted in Figs. 3-5. In these plots it can be observed that for large laser linewidths (BT=25), and as w is increased, significant performance gain is achieved compared to the conventional MFSK system.

Fig. 6 illustrates the average bit probability of error for MC(61,8,4) with BT=0, 10 and 25. This plot demonstrates the impact of increased phase noise on deteriorating the performance of the system. This agrees with behavior of the conventional MFSK system under similar conditions.

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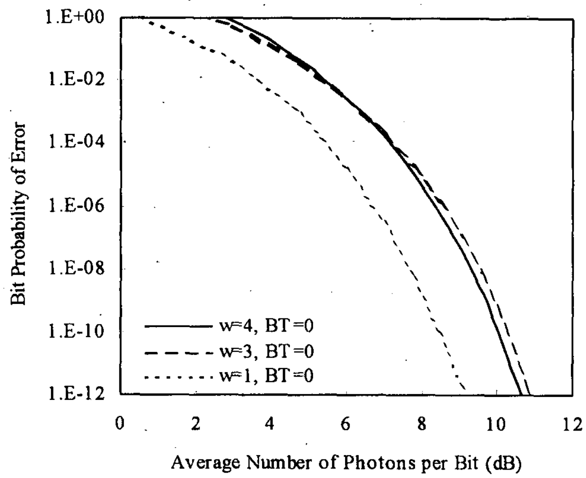

Fig. 3: Average bit error probability versus the average number of received photons per bit for the zero phase noise case (BT=0).

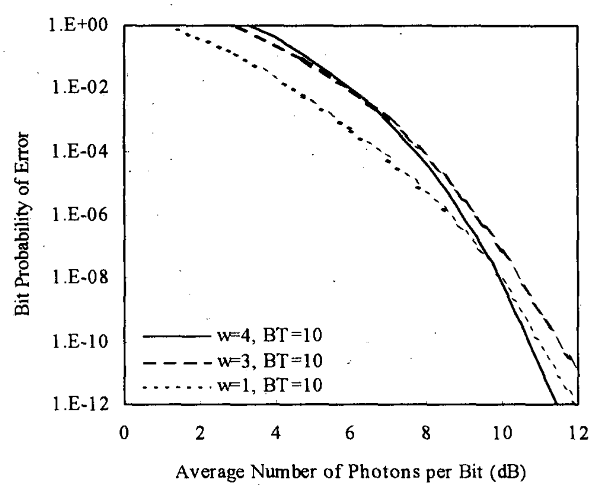

Fig. 4: Average bit error probability versus the average number of received photons per bit for medium linewidth-duration product (BT=10).

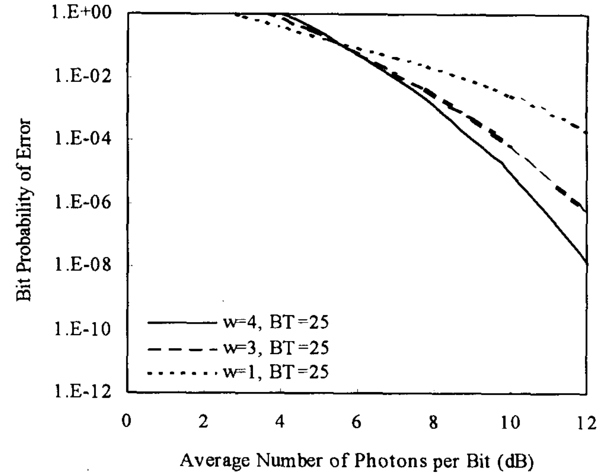

Fig. 5: Average bit error probability versus the average number of received photons per bit for large linewidth-duration product (BT=25).

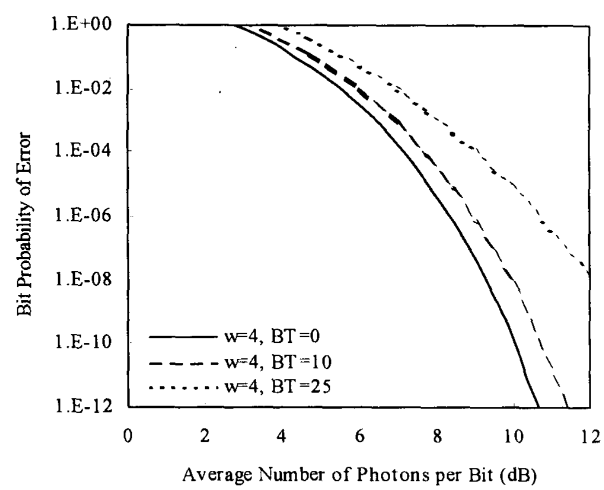

Fig. 6: Average bit error probability versus the average number of received photons per bit for w=4 and varying phase noise levels (BT=0, 10, 25).

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It should be emphasized that performance gains obtained in Figs. 3-6 are achieved while the number of tones used to construct the system are substantially reduced (see Table 1 above) compared to MFSK. This implies that much fewer lasers and wavelengths are needed.

V. CONCLUSIONS

A mathematical model of MC-MFSK signals based on BIB design has been presented. Such signals are characterized by their ability to increase the volume of a signal system without increasing the number of lasers required to represent the $M = 2^K$ symbols needed in the conventional MFSK system. This has the impact of reducing the system complexity while lowering the bandwidth required at a given bit rate.

The performance of MC-MFSK signals with laser phase noise and PD shot noise has been studied. A noncoherent receiver structure for MC-MFSK signals has been analyzed. A union bound expression for the bit error rate has been derived. The results show that MC-MFSK creates implicit frequency diversity of order w-1. Therefore, MC-MFSK performs better than the conventional MFSK system.

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