Reduction of Optical Beat Interference in SCM/WDMA Networks Using Pseudorandom Phase Modulation

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Abstract— A new approach is suggested to reduce the optical beat interference (OBI) in subcarrier multiplexed (SCM) wavelength-division multiple access (WDMA) networks. The idea is to deliberately introduce independent random polarization fluctuations in the electric fields transmitted on each optical channel. Random polarization results in an expanded OBI spectrum, and hence, less OBI power at the reference user receiving filter output. Electro-optic phase modulation is used to introduce polarization randomness in the fields before they are coupled into the fibers. A two-user system was simulated. Simulation results show the drastic reduction in OBI power spectral density using appropriate PN signals.

I. INTRODUCTION

S UBCARRIER multiplexing wavelength-division multiple access (SCM/WDMA) provides a simple approach to making use of the vast optical bandwidth available for communications (in excess of 10 THz). It is based on dividing the available optical bandwidth into separate optical channels each with a unique operating center frequency. After the available bandwidth is segmented into distinct optical channels, a set of modulated RF carriers intensity modulates each optical carrier. This is the subcarrier multiplexing step. A schematic diagram illustrating a hierarchy of different multiplexing schemes is shown in Fig. 1 [1]. There are N optical channels each modulated by M subcarriers.

The need for such a scheme arises from the following facts. On one hand, since the optical bandwidth is finite, using conventional WDMA without laser frequency stabilization would support a small number of optical channels each supporting a finite data rate [1]. On the other hand, reducing the channel separation would result in the so-called frequency-division multiple access (FDMA) which would make more channels available. Although FDMA looks attractive, it requires elaborate laser frequency referencing and stabilization. FDMA also requires coherent (heterodyne) optical detection or fine optical filtering. Such requirements make FDMA costly at the present state-of-the-art.

SCM/WDMA offers a cost-effective alternative to these schemes as a means to make use of the available optical bandwidth. Combined with direct detection, SCM/WDMA does not require complex circuitry for frequency allocation

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Fig. 1. A hierarchy of the multiplexing schemes.



Fig. 2. An SCM/WDMA system block diagram.

or stabilization. It does not require very sharp optical filters either.

A block diagram illustrating an SCM/WDMA system is illustrated in Fig. 2 As shown, a key component of the system is a passive star coupler. Although N wavelengths are used, the system supports communications links for MN users, as opposed to only N links in the case of conventional WDMA or FDMA. To send to user number k on optical channel number L, the sending party has to use optical carrier frequency F_L modulated by a subcarrier at f_k .

The problem addressed in this work is called beat noise or optical beat interference (OBI). To elaborate on causes and characteristics of this noise, let us consider the function of the photodetector (PD) in one of the user receivers. A total of M optical fields is present at the photodetector. Recalling the fact that in photodetection, the square of the optical field envelope over the photodiode is proportional to the detected photocurrent, we would expect as output M terms representing the field intensities, plus M(M-1) cross terms. If the spectral

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Fig. 4. Reference user receiver.

content of any one of these cross terms falls within the passband of the bandpass filter in the reference user receiver, it will cause interference.

Previous attempts have been made to reduce OBI. In [2] and [3], changing the laser direct intensity modulation index was employed. It was found that larger modulation indexes result in a lower OBI. However, there are certain limits beyond which the modulation index cannot be increased. Laser nonlinearity is an important factor in this respect. Besides, it is physically impossible to get an intensity modulation index larger than unity.

The proposed solution reduces the OBI using the idea of polarization randomization among the several fields coming into a single PD. This is achieved by phase modulating these fields by means of electro-optic modulators driven by different independent pseudorandom bias voltages. Phase modulation is performed on each field before it is coupled into the fiber. Random polarization results in an expanded OBI spectrum, and hence, less OBI power at the interference user receiving BPF output. However, the output power in the signal terms will not be changed; because of using direct detection which ignores the input field phase and polarization.

Fig. 3 is a schematic diagram of the so-called transverse electro-optic phase modulator. The bias voltage represents the PN modulating field. The voltage V_{π} required to induce π phase shift is given by [4]

$$V_{\pi} = \begin{cases} \frac{\lambda d}{n_x^2 r_{13}}; & \text{input beam } x\text{-polarized} \\ \frac{\lambda d}{n_x^2 r_{33}}; & \text{input beam } z\text{-polarized.} \end{cases}$$
(1)

Since for LiNbO₃, $r_{33} = 30.8 \times 10^{-12}$ m/V > r_{13} , the polarizer is set to transform input light into z polarization. In this case, assuming

$$d/l = 10^{-3}$$

and substituting in the second equality in (1), we get

$$V_{\pi} \approx 4.7 \text{ V}$$

which is a reasonable value for all practical applications.

The remainder of this paper is organized as follows. In Section II, the OBI problem is identified assuming all fields coming into a PD have the same polarization state. Reduction of the OBI using random polarizations is theoretically discussed in Section III. Simulation results also appear in Section III. Finally, in Section IV, the main results of this paper are summarized.

II. THE OBI PROBLEM

As shown in Fig. 1, M optical fields in a given optical channel have the same average center frequency. However, the simultaneous optical frequencies are generally different and can be varying with time. Each of these fields can be represented by

$$e_i(t) = [S_i(t)]^{1/2} \cos[\varphi_i(t)]$$
 (2)

where the intensity modulation by an RF subcarrier of center frequency f_i is represented by

$$S_i(t) = S_0 \{ 1 + m \cos\left(2\pi f_i t\right) \}.$$
 (3)

The phase of the electric field in (2) can have a component due to the operating optical frequency F_i , a chirp component $\varphi_{mi}(t)$ in the case of direct laser modulation, a phase noise component $\varphi_{ni}(t)$ modeled by a Wiener–Levy process, and possibly a random component $\varphi_{pi}(t)$ due to polarization fluctuation. Hence, the total phase can be written as

$$\varphi_i(t) = 2\pi F_i t + \varphi_{mi}(t) + \varphi_{ni}(t) + \varphi_{pi}(t). \tag{4}$$

The front end of the reference receiver (corresponding to user number k), as shown in Fig. 4, is an optical filter centered on the optical carrier frequency of the intended user channel. As a result, out of a possible total of MN optical fields coming into the optical filter, only M continue their way to the photodetector. The center frequency of the optical filter in the reference receiver will be denoted by F_0 . All M users in the same optical channel will have identical optical filters.

The total field e(t) at the input of the photodetector is the sum of M fields each of which can be written as in (2). Hence,

$$e(t) = \sum_{i=1}^{M} e_i(t).$$
 (5)

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The photodetector converts this field into an electric signal proportional to the field intensity. The constant of proportionality is equal to the photodetector responsivity. Since the same constant applies to all terms in the PD output current, it will be set to unity without loss of generality. This yields the (normalized) photodetector output as

$$I(t) = \mathrm{LP}\{|e(t)|^2\} = \mathrm{LP}\left\{\sum_{i=1}^{M} \sum_{l=1}^{M} e_i(t)e_l(t)\right\}$$
(6)

where the LP operator designates taking the low-pass part of the argument. The right-hand-side of (6) can be broken into a signal component $I_s(t)$ and a cross-term component $I_c(t)$ as follows:

$$I(t) = \text{LP}\left\{\sum_{i=1}^{M} e_i^2(t) + 2\sum_{i=1}^{M-1} \sum_{l=i+1}^{M} e_i(t)e_l(t)\right\}$$

= $I_s(t) + I_c(t).$ (7)

Note that, $I_c(t)$ constitutes an interference term that is generally nonzero. If any of the spectral components of $I_c(t)$ falls within the bandwidth of one of the M users' BPFs, it will cause OBI. To evaluate the resulting performance degradation, we need to calculate the signal and OBI power spectral densities. This objective is attended to in Section III.

III. REDUCTION OF OBI BY PHASE MODULATION

A. The OBI Spectrum

Let us assume, as in (2) and (4), that all fields have one polarization component which could be along any one of the two directions normal to the wave propagation direction. A more general polarization description may be used, but all it does is to complicate the analyses without adding real information about the system behavior. However, the individual field phases $\varphi_{pi}(t)$ will be assumed to be random.

The phase modulators in user channels are driven by a group of independent pseudorandom (PN) modulating signals. The clock rates of these PN signals are assumed to be equal and much higher than the subcarrier bandwidths. Assuming each PN signal changes its level in τ_0 seconds, and neglecting the chirp effect, the phase modulated field corresponding to the *i*th user can be represented by

$$e_{i}(t) = \operatorname{Re}\left\{ [S_{i}(t)]^{1/2} e^{j2\pi F_{i}t} e^{j\varphi_{ni}(t)} \sum_{k=-\infty}^{\infty} g(t-k\tau_{0}) e^{j\varphi_{pi,k}} \right\}$$
(8)

where g(t) is a unit amplitude pulse starting at time t = 0 and having a width τ_0 . The modulation phase $\varphi_{pi, k}$ is assumed to have one of the two possible values $\pm \varphi_p$ with equal probabilities, depending on the modulating field. Substituting (8) into (7), we get the signal and cross-term currents at the output of the photodetector as follows:

$$I_s(t) = \frac{1}{2} \sum_{i=1}^{M} S_i(t)$$
(9)

$$I_c(t) = \sum_{i=1}^{M-1} \sum_{l=i+1}^{M} I_{cil}(t)$$
(10)

where

$$I_{cil}(t) = [S_{il}(t)]^{1/2} \sum_{k=-\infty}^{\infty} g(t - k\tau_0) \\ \cdot \cos[2\pi F_{il}t + \varphi_{nil}(t) + \varphi_{pil,k}].$$
(11)

In the last equation, the following shorthand notation has been used

$$S_{il}(t) = S_i(t)S_l(t) \tag{12}$$

$$F_{il} = F_i - F_l \tag{13}$$

$$\varphi_{nil}(t) = \varphi_{ni}(t) - \varphi_{nl}(t) \tag{14}$$

$$\varphi_{pil,\,k} = \varphi_{pi,\,k} - \varphi_{pl,\,k}.\tag{15}$$

To evaluate the effect of phase modulation on the system performance, we need to calculate the signal-to-noise ratio at the output of a typical user bandpass filter. To do that, we first find the signal and the cross-term power spectral densities. When the PN clock rate is sufficiently high (much higher than the subcarrier bandwidth), contributions of phase noise and signal intensities to the cross-term PSDs can be neglected. Hence, in a typical cross-term, $\varphi_{nil}(t)$ will be omitted, and $S_{il}(t)$ will be approximated by a constant S_0^2 . On the basis of these approximations, the cross-term currents $I_{cil}(t)$ become statistically identical, in addition to being statistically independent. As a result, the power spectral density of the total cross-term current $I_c(t)$ can be approximated by the sum of the PSDs of the individual cross-term currents.

To simplify the calculation of the power spectral densities, the approximated $I_{cil}(t)$ will be written in the form

$$I_{cil}(t) \approx \operatorname{Re}\{I_{cil}(t)\}$$
(16)

where

and

$$\tilde{I}_{cil}(t) = S_0 e^{j2\pi F_{il}t} \tilde{P}(t)$$
(17)

$$\tilde{P}(t) = \sum_{k=-\infty}^{\infty} g(t - k\tau_0) d_k \tag{18}$$

$$l_k = e^{j\varphi_{pil,k}}.$$
 (19)

The power spectral densities of the last few quantities are related by [5]

$$S_{I_{cil}}(f) = \frac{1}{4} [S_{\tilde{I}_{cil}}(f) + S_{\tilde{I}_{cil}}(-f)]$$
(20)

$$S_{\tilde{I}_{cil}}(f) = S_0^2 S_{\tilde{P}}(f - F_{il})$$
(21)

and [6]

$$S_{\tilde{P}}(f) = \frac{1}{\tau_0} |G(f)|^2 \Phi_{dd}(f)$$
(22)

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where G(f) is the Fourier transform of g(t), and $\Phi_{dd}(f)$ is the PSD of d_k . Choosing $\varphi_p = \pi/2$, it can be seen that

$$\langle d_k \rangle = 0 \tag{23}$$

$$\langle d_k d_r^* \rangle_{k \neq r} = 0 \tag{24}$$

$$\langle |d_k|^2 \rangle = 1 \tag{25}$$

where the operator $\langle \rangle$ designates statistical expectation of the enclosed quantity. From (23)–(25), we immediately find that

$$\Phi_{dd}(f) = 1. \tag{26}$$

G(f) can be found from the definition of g(t) to be

$$G(f) = \tau_0 e^{-j\pi f \tau_0} \operatorname{sinc} (\pi f \tau_0).$$
(27)

Substituting (21), (22), (26), and (27) into (20), the *il*th cross-power spectral density is found to be

$$S_{I_{cil}}(f) = \frac{S_0^2 \tau_0}{4} [\operatorname{sinc}^2 (\pi (f - F_{il}) \tau_0) + \operatorname{sinc}^2 (\pi (f + F_{il}) \tau_0)]. \quad (28)$$

B. The Phase Modulation Advantage

To appreciate the phase modulation advantage in reducing the OBI, there are two noteworthy points at the beginning. First, the optical center frequencies were assumed fixed. This does not need to be the case in practice. However, it can be easily visualized that fluctuations in the optical center frequencies add further expansion to the OBI spectrum. This, in turn, results in less OBI PSD, and hence, less interference power into the reference use BPF. Second, the individual users PN signals used in phase modulation are generally asynchronous to one another. This is yet another source of the OBI spectrum expansion. This results from the fact that the phase difference $\varphi_{pi, k}$ stays only a fraction of τ_0 in one of its possible states.

Based on these facts, the assumptions of fixed optical center frequencies and synchronous PN modulating signal are worst case assumptions, since they give the least attainable OBI spectrum expansion. Carrying on with these assumptions, we add the simplifying assumption of equal optical center frequencies of all M fields coming into a photodetector. There is no loss of generality in introducing this assumption.

The OBI spectral density under equal optical center frequencies becomes

$$S_{I_{cil}}(f) = S_0^2 \tau_0 \operatorname{sinc}^2(\pi f \tau_0).$$
⁽²⁹⁾

Note that the terms in (28) add coherently when the optical carrier frequencies are equal. This results in equal PSDs for all OBI terms. Hence, to find the total OBI PSD, we simply multiply (29) by M(M-1)/2, the number of OBI terms. Assuming ideal bandpass characteristics of the reference (*kth*) user BPF, the output OBI power can be calculated as

$$P_{obi} = M(M-1)S_0^2 \tau_0 \int_{f_k - B/2}^{f_k + B/2} \operatorname{sinc}^2(\pi f \tau_0) \, df \qquad (30)$$

where B is the BPF bandwidth. Using an infinite series expansion of the integrand in (30), we get

$$P_{obi} = \frac{M(M-1)S_0^2}{\pi} \sum_{l=1}^{\infty} \frac{(-1)^{l+1}2^{2l-1}}{(2l)!(2l-1)} \cdot \left[\left[\pi \tau_0(f_k - B/2) \right]^{2l-1} - \left[\pi \tau_0(f_k + B/2) \right]^{2l-1} \right].$$
(31)

To get enough OBI spectrum expansion, the quantity $\tau_0 B$ must be as small as possible. For a high data rate transmission, f_k is a few multiples of B. Hence, for an effective OBI spectrum expansion, the terms $\pi \tau_0 (f_k \pm B/2)$ have to be very small. For example, if

$$\tau_0 \frac{B}{2} = 0.001$$

 $\tau_0 f_k = 0.005$

the second term in the summation in (31) is three orders of magnitude less than the first term. Hence, (31) will be approximated by keeping only the first term as follows:

$$P_{obi} \approx M(M-1)S_0^2 \tau_0 B \tag{32}$$

From (3), the signal PSD is

$$S_s(f) = \frac{S_0^2}{4} \left[\delta(f) + \frac{m^2}{4} \left[\delta(f - f_k) + \delta(f + f_k) \right] \right].$$
(33)

Consequently, the signal power at the BPF output is

$$P_{sig} = \frac{S_0^2 m^2}{8}.$$
 (34)

Dividing the quantities in (34) and (32), the signal-to-interference-ratio (*SIR*) is

$$SIR \approx \frac{m^2}{8M(M-1)\tau_0 B}.$$
(35)

To evaluate the system performance in terms of the probability of error, we neglect all other kinds of interference and noise, and assume that the spectrum-expanded OBI can be modeled as an AWGN. The last assumption can be justified by noting that the object is to increase the number of users M per optical channel by as much as possible, in which case, the central limit theorem can be used. We also assume binary antipodal signaling, i.e., the subcarrier signal in (3) is modified as:

$$S_i(t) = S_0 \{ 1 \pm m \cos(\omega_i t) \}.$$
 (36)

It is known that for binary antipodal signaling, the probability of error is [6]

$$P(e) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right) \tag{37}$$

where the signal-to-noise ratio γ is approximately equal to *SIR* in our case. To satisfy the 10^{-9} required probability of error, we solve (37) numerically in order to obtain a

$$\gamma \ge 18. \tag{38}$$

Substituting this result in (35) we get

$$\frac{1}{\tau_0 B} \ge \frac{144M(M-1)}{m^2}.$$
(39)



Fig. 5. Subcarrier and OBI spectra under no phase modulation.

If we let, for example

$$m^2 \approx 0.5$$

M=2,

the required PN signal should satisfy

$$\frac{1}{\tau_0 B} \ge 576.$$

Although this is a large number, possibly requiring a very high-speed PN signal, it should be recalled that several worstcase assumptions were used to derive this result. These assumptions include: fixed and equal optical frequencies, same field polarization states, and synchronous spreading sequences. Generally, these assumptions are not satisfied in practical situations. This implies that moderately fast PN signals can deliver much more OBI spectrum expansion than may be necessary.

C. Simulation Results

A two-user system was simulated using MATLAB. The two subcarriers had frequencies of 4 and 5 GHz, respectively. Each subcarrier was PSK-modulated by a digital binary source producing two equally likely symbols. The subcarriers then intensity modulated two laser fields with a common modulation index of 0.707. In all simulations, laser phase noise was neglected because the objective was to evaluate the spectrum expansion achievable by phase modulating the lasers with PN signals. The sampling frequencies used in simulations were equal to four times the rate of the respective PN signals. The signal and OBI spectra were estimated using a 1024-point FFT. In each of the figures about to be presented, the average power spectral density was computed using 100 runs of the respective simulation program. Parseval's theorem was applied to the estimated spectra, such that they ideally have to be equal to theoretical values.

Fig. 5 shows the subcarrier and the OBI spectra without phase modulation. To help identify each of these spectra, the two laser center frequencies were separated by 8 GHz. Figs. 6 through 8 show the subcarrier and OBI spectra around the subcarrier frequency range. The PN advantage in reducing the OBI PSD by increasing the PN phase modulation rate is obvious in these figures. Figs. 9 and 10 show the whole OBI spectra for two PN phase modulation rates.













It can be seen from the graphs in Figs. 6 through 10 that increasing the PN rate by 10 reduces the OBI PSD by 10 dB, which agrees with the theoretical predictions in (35).

IV. CONCLUSIONS

The problem of OBI in SCM/WDMA networks has been identified. The problem is due to the square-law envelope detection function of the photodetector. If no polarization control is employed, the OBI problem is expected to be present to an extent determined by the actual polarization states of the M optical fields coming into a PD.



Fig. 10. OBI Spectrum when $\tau_0 B = 0.001$.

Polarization randomization has been suggested as an efficient solution to the OBI problem. It has been shown that a dramatic reduction in the OBI at the desired user BPF output can be achieved with appropriate PN signals. Simulation of a typical two-user SCM system demonstrated the effectiveness of the proposed solution.

The so-called transverse electro-optic phase modulator is an excellent candidate to achieve the required polarization agility. This is because, unlike other phase modulators, electrooptic modulators do not use mechanical effects to adjust polarization. Electro-optic phase modulators are also compact in size and easy to integrate. The transverse modulator, in particular, needs a reasonably low bias voltage to induce the desired phase shifts.

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